Gravitational Acceleration: A case of constant acceleration

(approx. 2 hr.) (6/7/11)

Introduction

The gravitational force is one of the fundamental forces of nature. Under the influence of this force all objects having mass are attracted by the masses of other objects. This means that near the surface of the earth all objects are attracted by the mass of the earth itself, and will fall towards the center of the earth under the influence of gravity. Because the attraction is proportional to mass, all objects will fall with the same constant acceleration, which is denoted by the letter g.

Equipment

- spark timers (2/class)
- 500g mass & hanger
- scotch tape
- level
- thermal tape to record discharge pts.
- rods & clamps to hold timers
- masking tape (to hold thermal tape in place while measuring)
- vertical rod & bench edge clamp
- ruler & meter stick
- pad to protect floor
- graphing software
- graph paper (back up)

Note: just 2 setups at front of room for entire class

Before the Lab

Read the sections in your text describing one-dimensional motion and constant acceleration. Be familiar with the concepts of position, velocity, acceleration and the slope of a graph.

Theory

An object falling near the surface of the earth experiences a uniform (constant) acceleration. Because acceleration is constant, in a downward direction, and equal to g, its motion can be described by the equations:

$$y(t) = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

$$v(t) = v_0 + gt$$

Here we have defined "down" as being the positive y direction.

These equations predict that a plot of <u>velocity</u> versus time will start at the value v_o at time t=0 and show a straight line with positive slope. The value of that slope will be equal to acceleration (the rate of *change* of velocity, $a=dv/dt$). Thus we can measure the value of g by determining the slope of a graph that plots velocity versus time for a falling object.

Procedure

In this lab you will measure the free-fall acceleration of a mass. The free-fall apparatus consist of a "spark timer" and thermal tape. A "freely" falling mass attached to one end of the tape pulls the tape through the spark timer. Voltage pulses between the two electrodes of the spark timer leave marks on the conductive (i.e. shiny) side of the thermal tape. This records the changing position of the object as a function of time. The voltage pulses are applied 60 times per second, so the time between pulses is $\frac{1}{60}$ of a second.

Your lab instructor will make a tape for you or assist you in making one. (<u>WARNING</u>: STAND CLEAR OF IMPACT AREA TO PREVENT INJURY TO FEET!) Carefully examine the tapes and draw small circles around the spark marks so their locations are easily seen. (Do not cover the marks themselves, just outline them). Occasionally a spot in the sequence may be missing. If you think there is a missing spot, draw a question mark about where you think it should be.

Choose one spot near the top to be “point 0” where the position is y_o=0 at time t=0. This should not be the first spot. Do not choose the large spot where the object rested prior to being released! Choose a spot, that is easy to measure and where all the following points appear to travel in a nearly straight line, with no missing spots. (Note: Because this is not the “first” spot we cannot assume that the initial velocity, v_o, is equal to zero.

Using the data table (at the end of this handout) begin numbering points 0,1,2,3… and record times for each point so that point 0 is t_0=0, point 1 is t_1=1/60 sec=0.0167 sec, point 2 is t_2=2/60 sec =0.0333 sec etc. If there are missing spots you should include a position for them in your table and write “missing” beside these points.

Draw a straight line through each spot perpendicular to the edge of the tape. You will use these lines to measure the distance to each spot.

Carefully measure the distance from spot number zero to each following spot on your tape. Record these positions in your table as position y_i.

You will now compute the velocity at for each spot. At point “ i ” (where i means 1,2,3 etc.), the position is given by:

$$y_i = y_0 + v_0 t_i + \frac{1}{2} g t_i^{\,2} \qquad \text{(Equations 1)}$$
$$v_i = v_0 + g t_i$$

It can also be shown for constant acceleration only that the average velocity between two times is equal to the instantaneous velocity at a time halfway in between. Thus, for equally spaced time intervals, the instantaneous velocity at time t_2 is equal to the average velocities between times t_1 and t_3:

$$v_2 = \bar{v} = \frac{y_3 - y_1}{t_3 - t_1} \qquad \text{(Equation 2)}$$

Use Equation 2 to find the instantaneous velocity for points 1, 2, 3 etc. Use the positions and times for points 3 and 1 to calculate v_2, for points 2 and 4 to calculate v_3, for points 3 and 5 to calculate v_4, etc. Record each in your data table.

You can use a similar equation to calculate and record average accelerations:

$$a_2 = \bar{a} = \frac{v_3 - v_1}{t_3 - t_1} \qquad \text{(Equation 3)}$$

Make a graph of velocity versus time for your data. Your instructor will show you how to do this on the computer. If your graph makes it apparent that you failed to take into account missing points, adjust the times accordingly. (See Appendix E for notes if you are using Excel.)

Using the computer program again, fit a straight line to your data. The slope of this line will be your experimental value for g, which you can call “$g_{experimental}$”.

Determine the slope of your line and record it with your data.

Calculate and record the percent difference between the experimental value and the commonly accepted value, g=9.8 m/sec^2, using the equation:

$$\text{Percent difference} = \Delta g = \frac{g_{\exp erimental} - g}{g} \times 100\% \qquad \text{(Equation 4)}$$

If v_o is in fact not zero at t=0, then your graph will intercept the vertical axis at some velocity other than zero. Determine v_o from your graph.

Make a plot of position versus time using the computer. If position increases as time squared then this will be a *quadratic* curve. Use the quadratic curve fit to fit this data and find a second value for “g”.

For your report

Print copies of the graphs that you made for each student in your group. On each graph you should label the axes and record the slope and intercept. Write a cover page with a summary describing the experiment. Explain how you obtained the values of g and v_0 from your data. Compare the value for g to the accepted value and discuss possible sources of discrepancy. *You must explain why you expect your velocity graph to be a straight line and why its slope should be "g".*

Comment on how the value of "g" determined from your graphs compares to the variation in values calculated point-by-point in your table.

Suppose the time difference between sparks were not precisely 1/60 second. How would that affect your results? (Would $g_{\text{experimental}}$ be too large? too small? the same?)

Data Table: Free Fall with Spark Timer

Point i	Time from t=0, t_i	Distance from y=0, y_i	Calculated velocity, v_i	Calculated acceleration, a_i
0	0 sec	0 cm		
1				
2				

Remember measurements should include units!

Value of “*g*” from velocity graph, $g_{\text{experimental}}$ ____________________________

Percent difference from accepted value, %Δg ____________________________

Initial speed from velocity graph, v_o ____________________________

Value of “*g*” from position graph, $g_{\text{experimental}}$ ____________________________

Percent difference from accepted value, %Δg ____________________________

Initial speed from position graph, v_o ____________________________

ADDENDUM TO GRAVITATIONAL ACCELERATION LAB

ALTERNATIVE CALCULATIONAL INSTRUCTIONS (provided by Dr. Flores)

1. Make sure that no points are missing on the tape.
2. Fasten tape to table using masking tape.
3. Skip the first two points; the first point you use should be marked as x=0 cm (and t=0). Place a meter stick along the tape matching the first point used with 0 on the meter stick.
4. Read the positions of all other points using the meter stick and record in a table. Use about 20 consecutive points. Note that an error as small as a millimeter can result in a big fluctuation in the acceleration.
5. Make a table like the one below but with at least 20 points:

Time	x (cm)	Δx	v=Δx/Δt	Δv	a=Δv/Δt
0	0	2-0	2*60=120	180-120	60*60=3600
1/60	2	5-2	3*60=180	210-180	30*60
2/60	5	8.5-5	3.5*60=210	240-210	30*60
3/60	8.5	12.5-8.5	4*60=240		
4/60	12.5				

IMPORTANT

NOTE: at time t=0, position x=0, velocity v=120, and acceleration a=3600

6. Find the average and the deviation of the acceleration. Write your result as $a = \bar{a} \pm d_a$
7. Is your result in agreement with a=980 cm/s? If your acceleration in free fall is not in agreement with accepted value repeat experiment.
8. Plot using DataStudio x vs. t v vs. t a vs. t
    a. Note: if the value of R in your plots is 1 you have a perfect fit
9. Find the equation of the straight line in your v vs. t graph. Compare the slope with $\bar{a}$ in step 6.

\-----------------------------------

Sample calculation of average and deviation:

Given: 3,4,6,5,7,4 average: $\bar{a} = \left(\sum a_i\right)/N = (3+4+6+5+7+4)/6$ =4.8

The deviation:

$d_a = \left(\sum |a_i - \bar{a}|\right)/N = \left(|3-4.8| + |4-4.8| + |6-4.8| + |5-4.8| + |7-4.8| + |4-4.8|\right)/6$ =1.16

(the vertical bars mean absolute value)

Result: $a = 4.8 \pm 1.16$