Relating Force to Linear Motion: Discovering Newton's Second Law

(completion time: approx. 2 h 20 min.) (10/24/2017)

Introduction

A physical "law" is a fundamental principle that underlies our understanding of how the physical world works. Newton's second law of motion describes the relationship between the acceleration of an object, the mass of the object, and the net force acting on the object. In this lab you will use a simple system of pulleys, strings and weights to investigate this relationship for linear motion.

Equipment

- PASCO SMART Cart (0.25 kg)
- 2.2 m track with feet/end stops
- string with loops on ends (approx. 31 in.)
- PASCO Super Pulley
- level
- mass hanger (approx. 50g)
- 3 slotted masses (50g each)
- mass (metal, rectangular) (250g)

<u>For class</u>: a triple beam balance; spare string & scissors

Before the Lab

Read the sections in your text describing Newton's Laws. You should learn how to draw a force diagram and determine the vector sum of all the forces (i.e. the *net* force) on an object.

Newton's Second Law:

Based on careful observation and comparison of numerous measurements and calculations, Isaac Newton found that the acceleration of an object is directly proportional to and in the same direction as the net force on the object, and inversely proportional to the mass of the object: $a \propto \frac{F_{net}}{m}$. If the proportionality constant is chosen to be one, then this relationship becomes a vector equation that defines the unit of force in terms of the chosen units of mass and acceleration: $\vec{F}_{net} \equiv m\vec{a}$. If mass is measured in kilograms and acceleration in m/s/s, then the unit of force is called a Newton.

Experiment 1: Push and Pull

In this activity you will familiarize yourself with the force sensor and the acceleration sensor and verify Newton's second law for the case of linear motion:

$$F_{net} = ma.$$

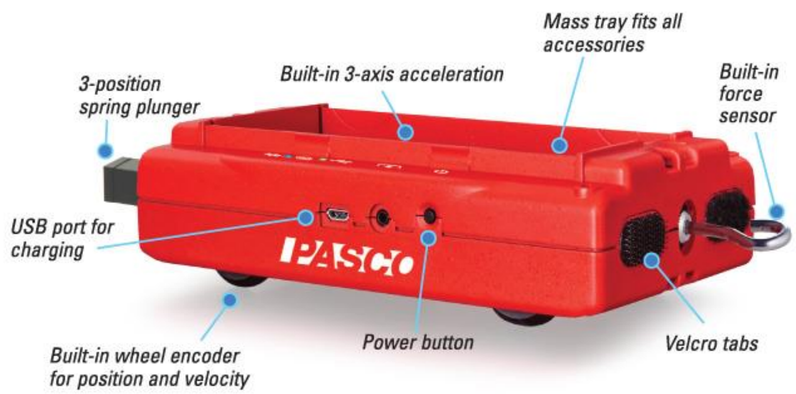
Computer and Experiment set-up:

- Set up the 1.2 m track and level it using a bubble level and the leveling screw on the end.
- Plug the USB Bluetooth dongle into the computer
- Launch PASCO Capstone from the desktop icon.
- To turn the Smart Cart on, press and hold the ON button for a moment until a status LED starts blinking. To turn the Smart Cart off, press and hold the ON button for a moment until the status LEDs stop blinking. The Wireless Smart Cart puts itself to sleep after several minutes of inactivity if not connected and a much longer time of inactivity if connected.

- For PASCO Capstone, select Hardware Setup in the Tools palette. The devices in the list are ordered by proximity to the tablet or computer. Select the Smart Cart with the address that matches the XXX-XXX Device ID number on the Smart Cart.
- Drag and drop two graphs in the work surface from the Display Palette.
- Create a force vs. time for the first graph.
- Create an acceleration vs. time for the second graph.

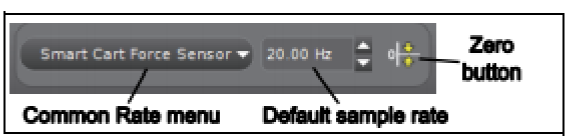
Testing and zeroing the sensors

With the cart at rest and the force sensor undisturbed start the program and let it run. You should see two horizontal straight lines for the values of force and acceleration.



IMPORTANT:

- At the beginning of an experiment, the measurement from the built-in force and acceleration sensors may not be zero when the force or acceleration is actually zero.
- Select Smart Cart Force Sensor from the Common Rate menu and then click the Zero button. This will set the sensor reading to zero.



- Test the acceleration sensor by abruptly moving the cart back and forth without touching the force sensor. Watch the acceleration graph change. Does the force change significantly?

Activity:

For this activity, you will push and pull the cart back-and-forth on the dynamics track with your hand touching only the hook of the force sensor. The acceleration sensor will measure the acceleration of the cart, while the force sensor will measure the force you exert on the cart hook.

1. The mass of a typical cart plus sensors is given as M = 250g. Record this value in the data table below.
2. Start a new run and record about 10-20 back-and-forth motions while alternately pushing and pulling the cart by the hook on the force sensor. One cycle should take about 1 s while covering a distance of about 25 - 40cm.
3. When you are satisfied that you have good plots of F vs. t and a vs. t on the screen, **Print** these graphs.
4. Now make a Force vs. Acceleration graph <u>using the same data Run(s)</u> as follows: Select Force for the y-axis and X-Acceleration for the x-axis. Take note of the relationship between the graphs. What is the “shape” of the graph? How does this relate to Newton’s Second Law of Motion? Sketch or print this graph for your lab notes.

Analysis and Further Measurement

Make a linear fit to the Force versus Acceleration.

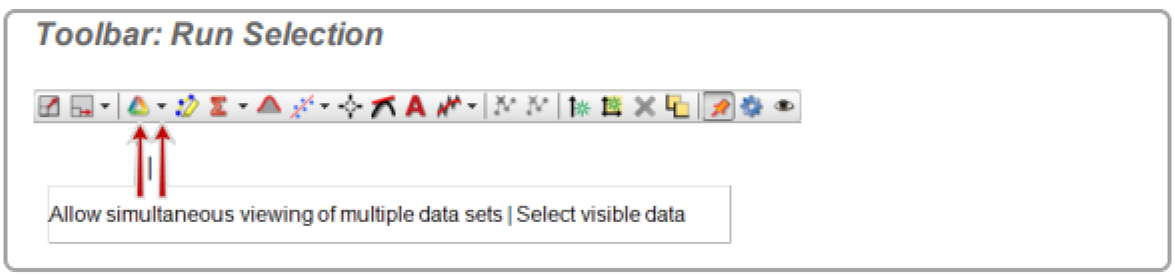
→ choose “Fit” from the Force vs. Acceleration graph menu and then “Linear”

Record the slope (m) on the data table below. What physical meaning of the slope of the line? How does this value compare to the cart?

→ Put the metal mass bar (250g) on the cart and repeat the experiment. Record the data on the same Force vs. Acceleration graph with the earlier run still visible. Repeat analysis.

When finding the slope for the new run, be sure to select the second run from your data **Print** the graph! See instructions for selecting multiple or single data sets (runs) below.

1. Click to depress **Run Selection** in the toolbar.



Selected runs display simultaneously.

2. Click the pull-down arrow (▾) next to **Run Selection** in the toolbar.
3. Select the one or more runs to display.

Data Table *(Remember, change units to kg!)*

<table>
<tr><td colspan="2">Cart with sensors:</td></tr>
<tr><td>Given Mass of Cart Plus Sensors:</td><td rowspan="2">Percent difference between given mass and measured slope:</td></tr>
<tr><td>Slope:</td></tr>
<tr><td colspan="2">Cart with sensors + 250g bar:</td></tr>
<tr><td>Given Mass of Cart Plus Sensors Plus 250g Bar:</td><td rowspan="2">Percent difference between given mass and measured slope:</td></tr>
<tr><td>Slope:</td></tr>
</table>

Questions:

1. How is the force related to the acceleration?

   Compare the minima and maxima of the force vs. time graph to the minima and maxima of the acceleration graph. How are they related?

2. Describe how you applied force to make the cart move back and forth?

   Draw sketches of the cart at the turning points and at the center of the track and show the direction of the applied force in each case.

3. At which point in the movement is the magnitude of the force maximal?

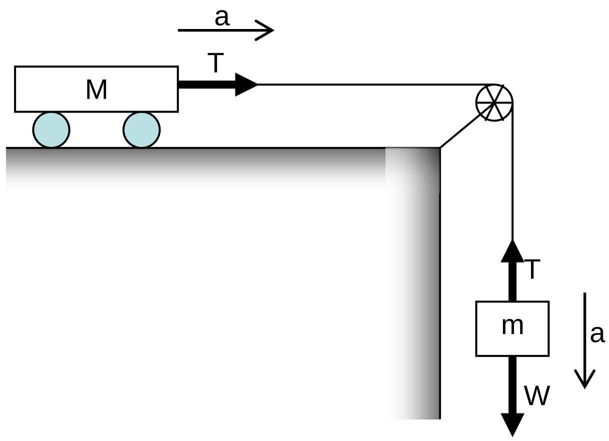
   At which point in the movement is the magnitude of the force minimal?

4. How do you interpret the force versus acceleration graphs for your measurements?

Experiment 2: Acceleration due to Hanging Mass

Theory

Consider two masses connected by a string over a pulley as shown. One mass, *M*, consists of a cart with sensors resting on a horizontal track. The other mass, m, is hanging freely and is subject to a downward force due to gravity (i.e. its weight): $W = mg$, and an upward force due to the tension T in the string. The masses of the string and pulley as well as the frictional resistance of the cart wheels and the pulley are assumed to be negligible.



<u>*Cart at Rest:*</u> When the cart is held stationary, there is no net force on the hanging mass, so the tension in the string is given by: $T = W$;

<u>*Cart accelerating:*</u> Since the length of the string does not change, the cart (with sensors) and the hanger accelerate at the same rate, *a*, (see figure).

→ From the **free body diagram of the hanging mass**, we get:

$$W - T = ma,$$ where *m* is the hanging mass.

→ Assuming the frictional and rolling resistance of the cart to be negligible,
a **free body diagram of the cart** yields:

$$T = Ma.$$ where *M* is the total mass of the cart and sensors.

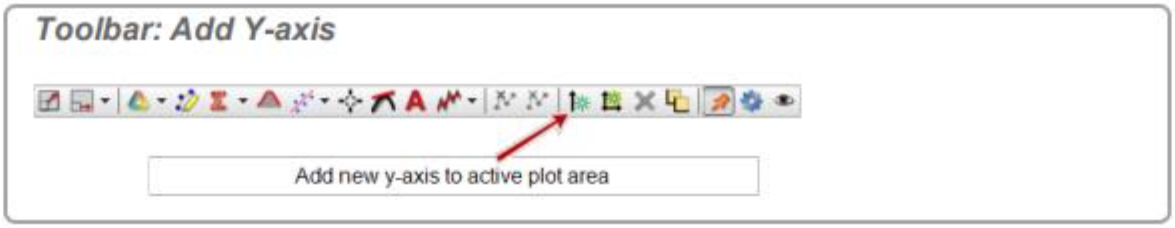
Required Derivation

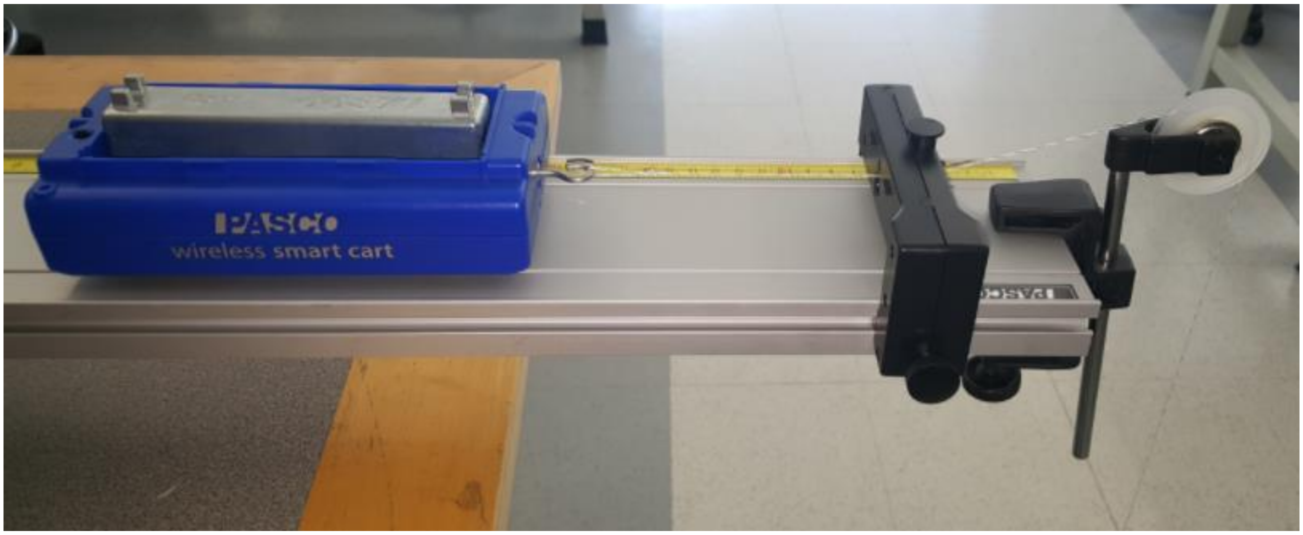
Combine the two preceding equations to express the tension and the acceleration as functions depending only on *m, M,* and *W*. These formulas will later be used to calculate the theoretical values for *T* and *a*.

$a =$ $T =$

Experimental and Computer Setup

1. Attach a pulley to track, as shown in the picture, making sure it is centered.
2. Attach a total of *~100g* as a hanging mass (50g hanger + 50g slotted mass) to the string. Be sure to run the string through the hole (string guide) in the end stop. Leave the end stop in place as this will prevent the cart and pulley from coming into contact during the experiment.
3. Create a graph for force vs. time and acceleration vs. time on the same timescale as follows:
   - Make a graph of force versus time.
   - Create an acceleration versus time graph ON THE SAME GRAPH by clicking add a new y-axis to active plot area. Select acceleration as a second variable. This procedure allows for the display of force and acceleration on the same graph.





Procedure

1. Move the cart (with sensors) to the far end of the track, away from the pulley.
2. Click on the Start button, then, about three seconds later, release the cart.
3. **Catch the cart before it smashes into the pulley!**
4. Click on the Stop button.

The graph will display force and acceleration. You will need this data for the analysis below.

Analysis

1. Using a triple beam balance, verify the value of the hanging mass "m" and record the value in the Data Table below. Also, record the given 500g total mass of the cart and sensors..
2. Determine the **tension T at rest**
   - In the force vs. time diagram, highlight the data points measured before release of the cart by clicking and dragging with the highlight range of data tool (4^{th} from the left in the menu) over the data points.
   - Obtain and record the *Mean* value of the force during this period by clicking on the statistics button (Σ) and "Show All".
3. Calculate and record the **weight *W* of the hanger** from $W = mg$.
4. Determine the **tension, *T*,** measured during acceleration:
   - Highlight the relevant data points on the force graph. Click on Σ, and *Show All*.
   - Calculate the *Mean* value of the tension during acceleration and record its magnitude (i.e. absolute value) in the Data Table.
5. Determine the experimental **acceleration *a***:
   - Highlight the relevant data points on the acceleration graph. Click on Σ and *Show All*.
   - Record the magnitude of the *Mean* value of your measured acceleration.
6. Calculate the values for Tension Force and acceleration from your previously derived equations.

NOTE: Be sure to print or sketch all graphs needed for your write-up before deleting them.

Additional Measurements

- Add the metal rectangular mass to the cart. Repeat measurements and analyze data. [NOTE: Mass of metal rectangular bar is given as 250g]
- Leave black bar on cart but add a 100g slotted mass to the hanger to a total of ~200g. Repeat the measurement and analysis of the data.

Data Table

Run 1 (use mass units of kg)

m, hanging mass: ***M***, total Mass of Cart and Sensors:	
T measured with cart at rest: ***W*** of mass hanger, calculated:	Percent difference:
T measured during acceleration: ***T*** during acceleration, calculated:	Percent difference:
a measured: ***A*** calculated:	Percent difference:

Run 2: Mass Added to Cart (use mass units of kg)

m, hanging mass: ***M***, total mass of cart, sensors and 500g bar :	
T measured with cart at rest: ***W*** of hanger, calculated:	Percent difference
T measured during acceleration: ***T*** during acceleration, calculated:	Percent difference
a measured ***a*** calculated	Percent difference

Run 3: Mass Added to Hanger (use mass units of kg)

m, hanging mass: ***M***, total Mass of Cart and Sensors:	
T measured with cart at rest: ***W*** of hanger, calculated:	Percent difference
T measured during acceleration. ***T*** during acceleration, calculated:	Percent difference
a measured: ***a*** calculated:	Percent difference

Questions

1. How do acceleration and force vary if the hanging mass changes but cart mass remains the same?

2. How do acceleration and force vary if the cart mass changes but hanger mass remains the same?

3. What are some possible reasons for any differences between the measured and calculated values?

4. In run 1, by how much is the force on the cart reduced when you release the cart?

   How is this value related to the acceleration?