The Force Table: Addition and Resolution of Vectors (approx. 2h) (6/7/11)

Introduction

Many of the concepts used in physics must be described by both a *magnitude*, or size, and a *direction*. Some of these quantities are displacement, velocity, force, and acceleration. We use a *vector* to represent these quantities. A vector can be represented graphically by an arrow whose length is proportional to its magnitude and which points in the desired direction. It can also be represented mathematically by giving the *components* of the vector along three perpendicular directions. In this experiment we will investigate methods of adding the vectors that represent forces. We will practice resolving the vectors into components. We will use a force table to experimentally observe the addition of different force vectors.

Equipment

- force table with 4 pulleys
- 4 sets of slotted masses
- protractor/ruler
- plane level
- colored pencils
- graph paper

For the class as a whole: Balance

Before the Lab

Read the sections in your text describing vectors and how to add them. You should learn how to add vectors both *graphically* (by constructing a triangle or parallelogram) and by resolving them into perpendicular *components* and then adding the components.

Theory

Vector Components:

When we wish to describe a quantity, which has both a magnitude (size) and direction, we can represent it with a *vector*. The vector can be described in terms of *components,* analogous to directions on a treasure map. To describe the direction of the treasure, we might say: it is 5 strides in a northeasterly direction, or we might describe it as lying 3 strides east and 4 strides north, as shown in Figure 1. Similarly the vector **A** can be broken into components A_x and A_y. We describe this vector in terms of a horizontal direction, x, and a vertical direction, y (as shown in the figure). The angle θ measures the angle from the x-axis to the vector, **A**. The relationship between the magnitude (or length) of **A** and its components are then

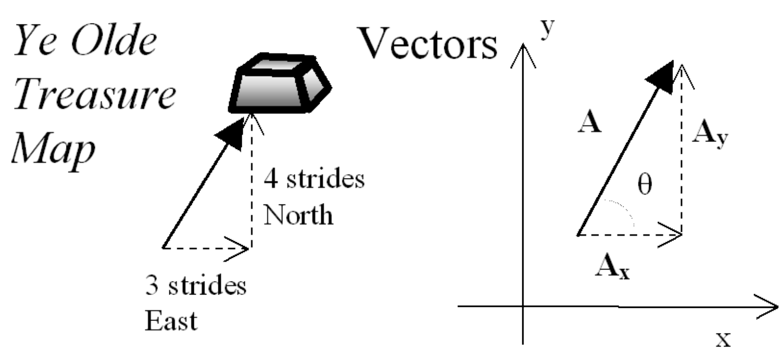

Figure 1: Components of a vector are analogous to directions on a treasure map.

$A_x = A\cos\theta$ where A_x is the side adjacent (or closest) to the angle

$A_y = A\sin\theta$ where A_y is the side opposite to the angle

For example if **A** is a vector 5 units long and the angle θ is equal to 53.1 degrees then we would find that

$$A_x = 5\cos\left(53.1^\circ\right) = 3$$
$$A_y = 5\sin\left(53.1^\circ\right) = 4$$

Thus **A** might represent the distance and direction to travel to find the treasure.
We can also find the magnitude and angle associated with **A** if we know its components, since

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

All of these relationships follow from the Pythagorean theorem since A, A_y and A_x form a right triangle.

Adding Vectors:

To add two vectors, **A** and **B**, to form a resultant vector, **C**, we can simply add their components to find the components of **C**, as shown on the right. Alternatively, the vector **C** can be found graphically by positioning **A** and **B** head to tail, as in the left side of Figure 2. **C** will be the vector that goes from the tail of **A** to the head of **B** and will be the diagonal of the parallelogram formed by **A** and **B**. The components of the vector **C** can also be found algebraically from the components of **A** and **B** as follows:

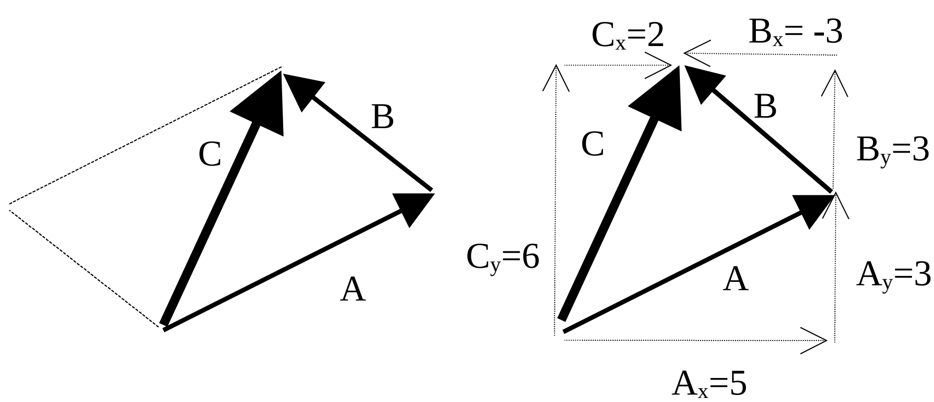

Figure 2: Addition of vectors. C is the vector sum of A and B. It can be found either graphically or by adding components.

$$\text{if} \quad \vec{C} = \vec{A} + \vec{B}, \quad \text{then} \quad \begin{cases} C_x = A_x + B_x \\ C_y = A_y + B_y \end{cases}$$

In Figure 2, **A=** A_x ***i*** +A_y ***j***=5 ***i*** +3 ***j*** and **B=** B_x ***i*** +B_y ***j***= -3 ***i*** +3 ***j*** then **C**=(5-3)***i***+(3+3)***j***=2***i*** +6***j*** or in other words C_x=2 and C_y=6. (Here the symbols ***i*** and ***j*** are used to denote the x and y directions, respectively.)

Vector Practice

Find x and y components of a vector which is 10 units in length and 40 degrees below the x axis (below the x axis is -40 degrees)

A_x= ___________________ A_y= ___________________

Check your results:

$\sqrt{A_x{}^2 + A_y{}^2} =$_________ $\tan^{-1}\left(\frac{A_y}{A_x}\right) =$__________

2. Find the components of a vector which has a length of 7 units and a direction 20 degrees above the x axis.

B_x= ___________________ B_y= ___________________

Check your results as above.

3. Find the result of adding **A** and **B** to find the component of **C = A + B**

C_x = ____________ C_y = _____________

Make a sketch showing the vectors A, B, and C and check the addition graphically (putting A and B head to tail and drawing a parallelogram).

Experimental Procedure

The *Force Table* is an apparatus used to determine the resultant (or vector sum) of different forces. Forces are applied radially to a central ring by means of attached strings, which run over pulleys on the edge of the table, with masses hanging on their ends. The pull of gravity on the masses (i.e. their weights: mg) gives rise to tension in the strings that is proportional to the amount of mass hanging. Therefore, the magnitude of a force may be varied by adding or removing mass. The direction of a force can be varied by moving a pulley along the circumference of the table. When two or more forces are applied to the ring, their vector sum, or resultant, can be found by finding the additional force needed to exactly balance the applied force. For example, if two forces are applied, the resultant, or vector sum, is

$$\vec{F}_1 + \vec{F}_2 = \vec{F}_{total}$$

the magnitude and direction of $\vec{F}_{total}$ may be found by finding a third force, $\vec{F}_3$, such that

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

When the net force on the ring is zero it will remain centered around the center pin, in *equilibrium*. The sum of $\vec{F}_1$ and $\vec{F}_2$ must then be equal in magnitude, but opposite in direction, to $\vec{F}_3$, i.e.,

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$$

In the following experiments you will practice different methods of adding vectors and then use the force table to experimentally check your calculations.

Begin by leveling the force table, if necessary. Then, practice balancing forces until you are able to determine when there is zero net force on the ring. Note that it is important that the strings tied to the ring slide easily from side to side, so that no sideways force is applied to the ring. The strings should pull straight outwards toward positions of the pulleys on the edges of the table. There will be some *uncertainty* in your experimental method. Remember that the weight of the hangers must be included in your total weights.

1) Given two force vectors, for example $\mathbf{F}_1$ corresponding to the weight of a 100 g mass at 30^0 (above the positive x-axis) and $\mathbf{F}_2$ corresponding to the weight of a 150 g mass at 140^0, find their resultant, or vector sum, by the following three methods. Record your results in your lab notebook, drawing appropriate diagrams to describe both your calculations and your experimental method.
   Graphically: draw a diagram to scale and construct a parallelogram to find the sum.
   Addition of components: on your diagram, define an x- and y-axis. Resolve the vectors into components along these axis (with your calculator) and find the sum. Does your answer make sense when you compare the result with the graphical method?
   Experimentally: Use the force table to determine the third force that would be required to balance the two force vectors previously defined. This force should be 180^0 opposite the force found by the previous two methods (since it cancels the first two forces). How does the result compare to you calculations. What is the *uncertainty* in your experimental result?

2) Repeat all of the above with two different forces in different directions, e.g. 100 g at 20^0 and 75 g at -80^0. This time try the experimental method first. Set up your two forces, then pull on the string to find the direction that a third force must be applied to balance the first two. Add weights to determine the third force. Then check your result by graphical methods and by addition of components. How do your results compare? Is this within the expected uncertainty of the experimental method?

3) Repeat using two perpendicular forces. You can choose the x- and y-axis to be in the perpendicular directions: for example, $\mathbf{F}_1 = F_x$ corresponding to 75 g at 0^0 , and $\mathbf{F}_2 = F_y$ corresponding to 100 g at 90^0.

4) You can also find the components of a vector experimentally. Place three pulleys at 240^0, 90^0 and 0^0. Hang a total of about 150 g from the string through the pulley at 240^0. You should now be able to find its components along the x-axis (at 0 degrees) and the y-axis (at 90 degrees) by finding the weights that you must hang from these pulleys to balance the weights. Check your results graphically and mathematically.

Questions

Compare the graphical and analytical (addition of components) methods for adding vectors. Which is more accurate? Give possible sources of error for both methods. Why is it useful to use both methods?

What are the possible sources of error in the experimental method? (Why is it necessary to allow the strings to slip loosely about the ring.)

If the weights of all the mass hangers were the same, could their weights have been neglected? Explain.

What is the effect of the weight of the ring? What difference would it make if the ring were considerably more massive?