

## Newton's Second Law – Atwood's Machine

### Equipment:

#### Included:

1	Photogate/Pulley System	ME-6838A
1	Mass and Hanger Set	ME-8979
1	Universal Table Clamp	ME-9376B
1	60-cm Long Threaded Rod	ME-8977
1	Multi Clamp	ME-9507
1	Braided Physics String	SE-8050
Required (Not included):		
1	850 Universal Interface	UI-5000
1	PASCO Capstone	UI-5400
1	Balance or Scale	SE-8707
1	Calipers (suggested: Digital Calipers)	SE-8710

### Introduction:

The purpose of this activity is to study the relationship between net force, mass, and acceleration as stated by Newton's 2<sup>nd</sup> Law, using an Atwood's Machine apparatus, built with a PASCO Super Pulley. The Super Pulley has very low friction and small mass.

The Photogate Head, when attached to a Super Pulley, is used to measure the velocity of both masses as one moves up and the other moves down. The slope of the graph of velocity vs. time is the acceleration of the system. Careful measurement when there is no net force allows the student to compensate for friction.

The effect of the motion of the pulley on the measured experimental acceleration will also be examined. This will provide a way to roughly compensate for the (small) effect of the pulley.

## Simplified Theory:

The acceleration of a system is directly proportional to the net applied force and inversely proportional to the system's mass, as stated by Newton's 2<sup>nd</sup> Law of Motion:

$$a = F_{\text{net}}/M_{\text{system}} .$$

Atwood's Machine consists of two unequal masses connected by a single string that passes over an ideally massless and frictionless pulley as in Figure 1. When released, the heavier object accelerates downward while the lighter object accelerates upward.

The free-body diagrams below show the forces acting on each of the masses.  $T$  is the tension in the string, assumed to be the same for both masses. This is a good assumption as long as it is a single strand of string and a relatively light pulley system. With  $m_1 > m_2$ ,  $m_1$  is the descending mass and  $m_2$  is the ascending mass. The magnitude of the acceleration,  $a$ , is the same for each mass, but the masses accelerate in opposite directions. We adopt the convention that down is positive. Then the equation of motion of the descending mass,  $m_1$ , is

$$m_1g - T = m_1a .$$

The equation of motion for the ascending mass,  $m_2$ , is

$$m_2g - T = m_2(-a) .$$

Eliminating  $T$  between the two equations yields:

$$(m_1 - m_2)g = (m_1 + m_2)a$$

Which may be solved for the acceleration

$$a = (m_1 - m_2)g / (m_1 + m_2) = F_{\text{net}}/M_{\text{system}} \quad \text{Eq. (1)}$$

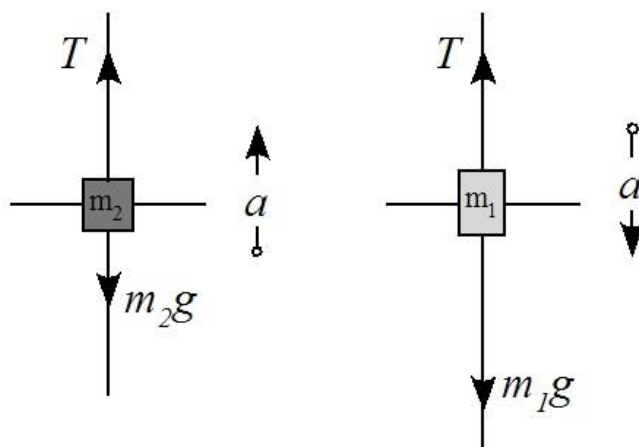
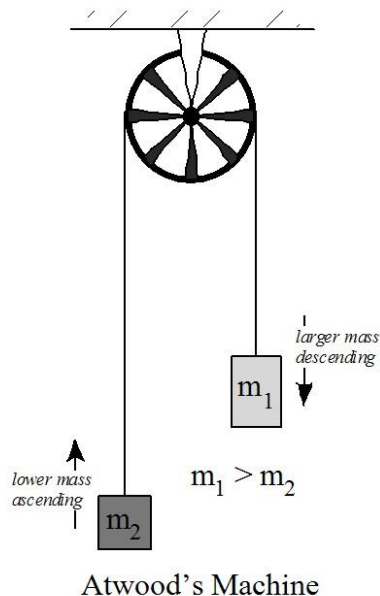


Figure 1: Atwood's Machine

Figure 2: Free Body Diagram for Atwood's Machine

## Setup:

1. Attach the PASCO Super Pulley to the Photogate using the 15 cm treaded black rod as shown in Figure 3. Attach one end of the photogate wire to the telephone jack on the photogate and the other end to Digital Input 1 on the 850 Universal Interface.
2. Attach the Multi Clamp to a table and arrange the rods as shown in Figure 4.
3. Cut 1.5 m piece of the braided string and tie loops on each end to hold a mass hanger. Determine the mass of the string in grams and enter it in column 1 (String Mass) of the Acceleration Data table under the Analysis tab. Enter the value in each of the first four rows.
4. Add a single 50 g mass to one mass hanger (5 g) for a total mass of 55 g. We will call this mass hanger  $m_1$ . To a second mass hanger (call it  $m_2$ ) add one 20 g, one 10 g, two 5 g, two 2 g, and one 1g mass for a total of 50 g (including the 5 g hanger).
5. Adjust the height of the pulley so that when mass hanger  $m_1$  is touching the floor, mass hanger  $m_2$  is a few centimeters below the pulley. The Photogate should be horizontal so the string does not pull sideways on the pulley.

## Friction & String Mass Compensation:

1. Add an additional 5 g to  $m_2$ . The two masses should now balance. If there were no friction and the string were massless, then if we gave  $m_1$  a push downward, it would continue at constant speed.
2. Move  $m_1$  to its highest point. Click RECORD. Give  $m_1$  a gentle push downward and release it.
3. When  $m_1$  stops moving or strikes the floor, click STOP. The masses may touch as they pass each other. If so, click the Delete Last Run button at the bottom of the screen and repeat the run.
4. Click the Data Summary button at the left of the screen. Double click on the current run (probably Run #1) and re-label it "Equal mass run". Click Data Summary again to close the panel.
5. Your Speed vs Time graph should look like Figure 5. Answer Question 1 on the Conclusions page.
6. Add 0.5 g to  $m_1$  and repeat. Label this run "0.5 g run". Then add another 0.5 g and repeat again until you get a graph like Figure 6. Since you only have 0.5 g increments, your graph may not be quite as symmetric as Figure 6. (Of course, you could add small paper clips to get as close as possible.) Note that the slowing/speeding up is now in the hundredths of a meter per second rather than tenths of a meter per second. Enter the amount of mass (in grams) you added to compensate for friction in column 2 (Friction Mass) of the Acceleration Data table under the Analysis tab in all four rows. Answer question 2 on the Conclusions page.
7. *Leave the extra mass on  $m_1$  to compensate for friction.* We will include the extra mass (+ any paper clips) and the mass of the string in the total mass of the system, but will not include them in the mass difference,  $m_1 - m_2$ .
8. **Remove the extra 5 g from  $m_2$ !**

**Procedure:** (sensors set at 10 Hz)

1. Transfer 2 g to  $m_1$  from  $m_2$ . Note that this keeps the total mass constant.
2. Move  $m_1$  to its highest point. Click RECORD. Give  $m_1$  a gentle push downward and release it.
3. When  $m_1$  strikes the floor, click STOP. It is better if you can catch  $m_2$  just before  $m_1$  hits the floor. Otherwise, the  $m_1$  hanger may come loose and you'll end up chasing masses around the floor. The masses may touch as they pass each other. If so, click the Delete Last Run button at the bottom of the screen and repeat the run.
4. Click the Data Summary button at the left of the screen. Double click on the current run and re-label it "55v50 g run".
5. Transfer 2 g from  $m_2$  to  $m_1$ . Note that this keeps the total mass constant. Repeat steps 1-3. Label this run "57v48 g run".
6. Transfer another 2 g from  $m_2$  to  $m_1$  and repeat. Label this run "59v46 g run".
7. Transfer another 2 g from  $m_2$  to  $m_1$  and repeat. Label this run "61v44 g run".

## Data:

1. Click on the black triangle by the Run Select icon on the graph toolbar and select “55v50 g run”.
2. Click the Scale to Fit icon on the left of the graph toolbar.
3. Click the Selection icon and adjust the handles on the selection box to highlight the linear portion of the data. Use as much of the data as possible but be sure not to include any points where the masses were not moving freely.
4. Click the black triangle by the Curve Fit icon and select “Linear”. The slope on the linear portion of a speed versus time plot is the acceleration. Record the value in column 8, row 1 of the Acceleration Data table under the Analysis tab.
5. Repeat for the “57v48 g run” data and record it in row two. Then do the other two runs.

## Analysis:

1. Click open the Calculator at the left of the screen. Examine line 3 and verify that the calculation of “Theory a” in column 7 agrees with Equation 1 from Theory.
2. Compare the “Theory a” column to the “Exp. a” column. The “Theory a” numbers should be consistently high. The percent by which “Theory a” is high is calculated in column 9 (“% high”).
3. What could be the source of the systematic error? If you answered “the pulley”, then you are correct. The pulley also has mass, and it accelerates. We will deal with rotating systems later in the course. For now we just assert that the pulley may be treated as an additional constant mass added to the total mass of the system. The effective total mass of the pulley wheel (which does not equal its mass) may be estimated from our four runs.
4. Calculate the average percentage by which “Theory a” is high (the average of the “% high” column).
5. Multiply the percentage (divided by 100) by the “Total mass” in column 5. Enter the value in all four rows of column 10 (“Effect. Rot. m”).
6. The corrected total mass (“Corr Tot m”), corrected acceleration (“Corr a”), and the percent by which the experimental a (“Exp. a) disagrees with the corrected acceleration (“% diff”) now show in last three columns.

## Conclusions:

1. For the “Equal mass run”, step 5 from the Friction tab:
  - a. Why are the masses slowing down?
  - b. Why is the rate at which the masses slow down decreasing with time (the slope is less negative)? Hint: does the string weight tend to speed the masses up or slow them down, or both?
2. For the run where the speed stayed as constant as possible, step 6 under the Friction tab:
  - a. Why do the masses first slow down and then speed up? Hint: remember the string.
  - b. You may not see the periodic oscillation that shows in Figure 6. If you do see it, try to explain it.
3. Does your data support Newton's Second Law of Motion? Support your answer!
4. The PASCO specs on the pulley wheel list its actual mass as 5.5 g. If all the mass were at the same distance as the string is from the axis of the wheel, the effective rotational mass would be 5.5 g. Some of the mass is actually further from the axis which increases the effective mass, but quite a bit of the mass is closer to the axis. The true effective mass is probably  $4.5 \pm 1.0$  g. How well does this agree with your results?

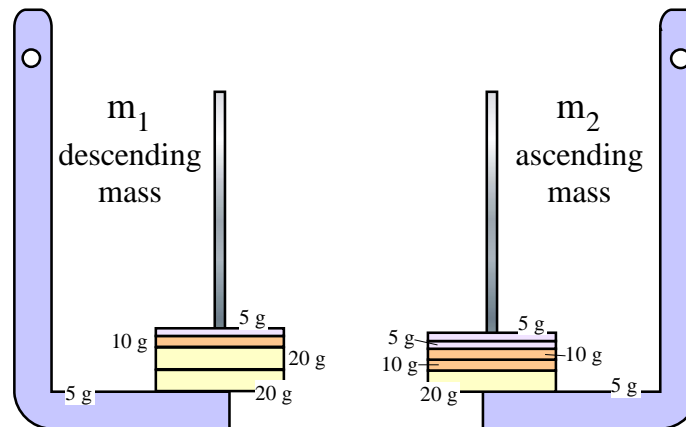


8.

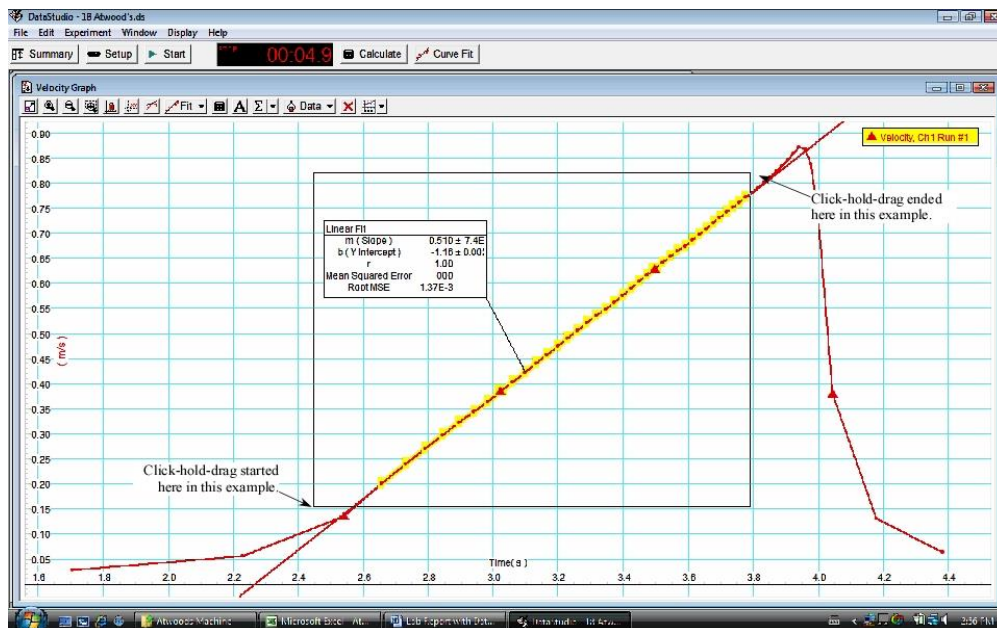
**PROCEDURE A: Total Mass Constant**

NOTE: The procedure is easier to perform if one person handles the apparatus and a second person handles the computer.

1. Place the illustrated combination of mass-pieces on the hangers. Record the masses  $m_1$  and  $m_2$  in the Data Table, including the 5-g mass of the hanger itself.



2. Move the  $m_1$  mass hanger upward until the  $m_2$  mass hanger almost touches the floor. Hold the string to prevent the system from moving just yet.
3. Click 'Start' to begin recording data and then release the string.
4. Stop the motion by gently grabbing the string, before the hangers reach the pulley at the top or the floor at the bottom. This will prevent the small pieces from jumping out of the hangers.
5. Click 'Stop' to end the data recording. Do not worry if the data on the graph has sections of "extra data" from the moments when the masses were still not moving, or if they collided with each other, or from after you stopped them.
6. Repeat two more times with the same masses  $m_1$  and  $m_2$ .
7. In the computer: Select Run #1 from the Data Menu in the Graph display. (If multiple data runs are showing, first select No Data from the Data Menu and then select Run #1.) Click the "Scale-to-fit" button to rescale the Graph axes to fit the data.
8. Click the 'Fit' menu button and select 'Linear'. The computer will make a linear fit of all the data and tell you the slope "m" of the best-fitting line.
9. If there are parts of the plot that are not needed, place the cursor on the lower-left edge of the "good data." Click, hold, and drag the cursor up until only the part of the plot needed is highlighted. (See the diagram below.)

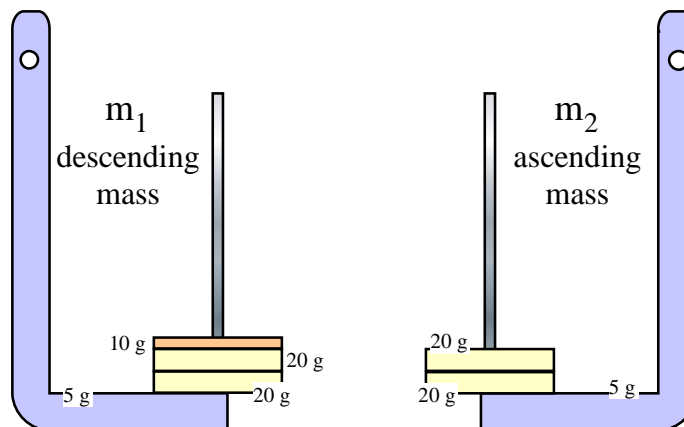


By highlighting only a portion of the graph, the linear fit calculation uses only those points.

10. In a graph of Velocity vs. Time, the slope is the acceleration. Record the value of the slope as the “Experimental Acceleration” in the Data Table.
11. Repeat for each of the data runs and then calculate the average experimental acceleration for this set of masses  $m_1$  and  $m_2$ .
12. For the next set of data, move a mass-piece from the  $m_2$  hanger to the  $m_1$  hanger. This will make the descending mass larger than before, the ascending mass smaller than before, while still keeping the total mass of the system unchanged. (This process changes the net force by changing the difference in weight of the hanging masses, but without changing the total mass in the system.) Record the new masses  $m_1$  and  $m_2$  in the Data Table. Repeat the data collection process.
13. Repeat the above step to create two more mass combinations. For each run, the net force changes but the total mass of the system remains constant. Measure the experimental acceleration three times for each mass combination to get a good average.

**PROCEDURE B: Net Force Constant**

1. Place the illustrated combination of mass-pieces on the hangers. Record the masses  $m_1$  and  $m_2$  in the Data Table, including the 5-g mass of the hanger itself.



2. Add 5 grams to each mass hanger. Record the new  $m_1$  and  $m_2$  in the Data Table. Notice that the addition of the extra mass only changed the total mass of the system, but without altering the difference between the ascending and descending masses. Record data and measure the experimental acceleration three times to calculate an average
3. For the next trial, place more mass on each hanger, the same amount on each. This will increase the total mass of the system, but without changing the difference between  $m_1$  and  $m_2$ .
4. Measure the acceleration of each mass combination three times to calculate the average.
5. If you want to perform Procedure C, the examination of the effect of the pulley, dismantle the equipment and use a scale to measure the mass of the string. Record it in the space provided in the Data Table for Procedure C.

**PROCEDURE A & B: Calculations**

1. For each of the data runs, calculate the total mass  $m_1 + m_2$  and record it in the Data Table.
2. For each of the data runs, using the measured mass values, calculate and record the net force:  $F_{net} = (m_1 - m_2)g$
3. Using the total mass and the net force, calculate the theoretical acceleration using Newton's 2<sup>nd</sup> Law of Motion:

$$a = \frac{F_{net}}{M_{total}}$$

4. For each data run, calculate and record the percent difference between the experimental acceleration and the theoretical acceleration:

$$\% \text{ difference} = \frac{|a_{\text{exp}} - a_{\text{theo}}|}{\left(\frac{a_{\text{exp}} + a_{\text{theo}}}{2}\right)} \times 100$$

(The percent difference compares the difference between the values to the average of the values.)

## PROCEDURE C: The Rotational Inertia and the Friction of the Pulley System

### THEORY

#### *Theoretical Assumptions*

One of the main assumptions of the Atwood's Machine theoretical analysis is that the pulley-system has no significant mass and that its motion has no effect on the measured acceleration of the system. However, it is obvious to the observer that the pulley moves together with the hanging masses and is thus also part of "the system." (So is the string, by the way.) In order to account for this "extra mass," let's introduce a term to the total mass of the system:

$$M_{\text{total actual}} = m_1 + m_2 + m'.$$

Here  $m'$  will be the excess mass unaccounted for in the previous theoretical analysis. We expect  $m'$  to be (hopefully) very small.

The other main assumption is that the system is frictionless. This is also an idealization. Even for the specially designed type of bearing of the pulley there may be a small amount of friction that may drag the pulley. The strings may also contribute to the drag if they slip and rub against the pulley. The presence of any frictional force will reduce the net force.

Let's introduce a general frictional term to the calculation of the net force:

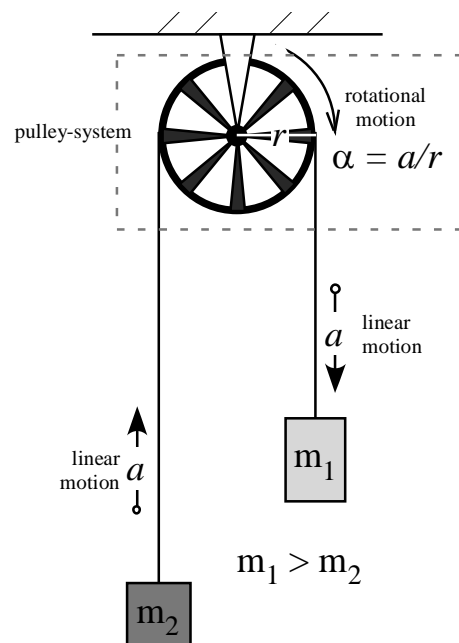
$$\begin{aligned} F_{\text{net actual}} &= F_{\text{net assumed}} - f' \\ &= (m_1 g - m_2 g) - f' \end{aligned}$$

Here  $F_{\text{net assumed}}$  is the net force we used in the ideal case, the difference in weight between the masses  $m_1$  and  $m_2$ .

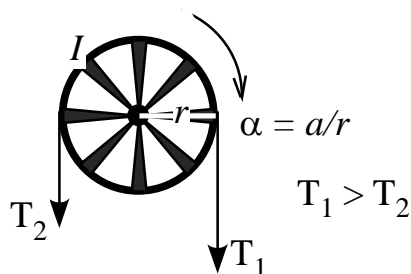
Notice that the presence of any excess mass and the presence of any frictional force must result in a measured value of acceleration that is lower than the ideal theoretical value predicted in procedures A & B.

#### *The Effect of the Rotation of the Pulley*

The pulley rotates with constant angular acceleration  $\alpha$ . As long as there is no slippage, the linear acceleration of any point on the rim of the pulley must be the same as the linear acceleration of the masses and of the string. That is,  $\alpha = a/r$ .



The body diagram below shows the tensional forces acting on the pulley, of radius  $r$  and moment of inertia  $I$ . No friction is considered in this part of the analysis.



Let  $T_1$  be the tension from the side of the string connected to  $m_1$ , and let  $T_2$  be the tension from the side of the string connected to  $m_2$ . These tensional forces provide the torques that make the pulley rotate. Because  $T_1$  tries to make the pulley rotate clockwise, while  $T_2$  tries to make the pulley rotate counterclockwise, the torques they apply are in opposite directions, but the torque from  $T_1$  must be greater, since we know the system in the end rotates and accelerates clockwise (in the direction of the larger, descending mass).

The equation of rotational motion for pulley is:

$$T_1 r - T_2 r = I \alpha.$$

Applying the no-slippage condition, this equation becomes:

$$T_1 r - T_2 r = I \frac{a}{r}$$

Now, from previous analysis (see the ‘Theory’ section for Procedure A) we know that  $T_1 = m_1 g - m_1 a$ , and that  $T_2 = m_2 a + m_2 g$ . Substituting into the previous equation, then simplifying and re-arranging, we see that the linear acceleration of the system must be:

$$a = \frac{m_1 g - m_2 g}{m_1 + m_2 + \frac{I}{r^2}} = \frac{F_{\text{net assumed}}}{m_1 + m_2 + \frac{I}{r^2}}.$$

If the pulley-system has any significant rotational inertia, then its mass contribution ( $m'$ ) to the system has the form  $m' = I/r^2$ . Notice that if  $I$  is very small this expression reduces to the theoretical formula used in the analysis of Procedures A and B. If there is also friction opposing the motion of the pulleys, the same analysis can be used to show that the effect is a reduction to the net force.

$$a = \frac{F_{\text{net (assumed)}} - f'}{m_1 + m_2 + \frac{I}{r^2}}.$$

### The Analysis

The experimentally measured acceleration must include all effects of excess mass and friction. Let's say that the measured acceleration for each run is actually given by

$$a = \frac{F_{\text{net assumed}} - f'}{M_{\text{total assumed}} + m'}$$

Rearranging this equation,  $F_{\text{net assumed}} - M_{\text{total assumed}} a = m' a + f'$ , where  $m'$  and  $f'$  are the unknowns we are attempting to find.

### PROCEDURE C: Calculations

1. Transfer the following data from Procedures A and B into the Data Table for Procedure C: the measured experimental acceleration of each run (this will be  $a$ ), the net force of each run (this will now be  $F_{\text{net assumed}}$ ), and the total mass of each run (this will now be  $M_{\text{total assumed}}$ ).
2. Calculate the quantity  $F_{\text{net assumed}} - M_{\text{total assumed}} a$  for each data run and enter the results in the table.
3. Make a plot of the quantity  $F_{\text{net assumed}} - M_{\text{total assumed}} a$  in the y-axis versus the experimental acceleration,  $a$ , in the x-axis. Notice that the equation should be linear:

$$\begin{array}{ccccccc}
 F_{\text{net}} & - & M_{\text{total}} & a & = & m' & a & + & f' \\
 \text{assumed} & & \text{assumed} & & & & & & \\
 \downarrow & & & & & \downarrow & \downarrow & & \downarrow \\
 y & & & & = & m & x & + & b
 \end{array}$$

The excess mass  $m'$  will be the slope of the line. The frictional force  $f'$  will be the y-intercept.

4. Draw the best-fitting line for your plot and calculate the slope and the y-intercept. Record the values as  $m'$  and  $f'$ , respectively. Don't forget the units.
5. Use calipers to measure the diameter of the Super Pulley. Determine the radius of the pulley and enter it in the Data Table.
6. Use the calculated excess-mass  $m'$  to calculate the rotational inertia of the pulley:

$$I = r^2 m'$$

### Lab Report: Newton's Second Law – Atwood's Machine

Name: \_\_\_\_\_

#### DATA TABLE for PROCEDURE A: Total Mass Constant

Run	Descending $m_1$	Ascending $m_2$	Experimental $a_{exp}$ (Slope of the $v$ vs. $t$ plot.)			
			Trial 1	Trial 2	Trial 3	Average
1						
2						
3						
4						

[g]                      [g]                      [m/s<sup>2</sup>]

Run	Descending $m_1$	Ascending $m_2$	Total Mass $M_{total}$ $(m_1 + m_2)$	Net Force $F_{net}$ $(m_1 - m_2)g$	Experimental $a_{exp}$ (Average)	Theoretical $a_{theo}$ $F_{net} / M_{total}$	Percent difference
1							
2							
3							
4							

[g]              [g]              [g]              [N]              [m/s<sup>2</sup>]              [m/s<sup>2</sup>]



**DATA TABLE for PROCEDURE B: Net Force Constant**

Run	Descending $m_1$	Ascending $m_2$	Experimental $a_{exp}$ (Slope of the $v$ vs. $t$ plot.)			
			Trial 1	Trial 2	Trial 3	Average
5						
6						
7						
8						
	[g]	[g]	[m/s <sup>2</sup> ]			

Run	Descending $m_1$	Ascending $m_2$	Total Mass $M_{total}$ $(m_1 + m_2)$	Net Force $F_{net}$ $(m_1 - m_2)g$	Experimental $a_{exp}$ (Average)	Theoretical $a_{theo}$ $F_{net}/M_{total}$	Percent difference
5							
6							
7							
8							
	[g]	[g]	[g]	[N]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	

**DATA for PROCEDURE C: The Effect of the Pulley**

Mass of string used in Procedures A & B: \_\_\_\_\_ g

Diameter of the Super Pulley: \_\_\_\_\_ cm

Radius of the Super Pulley,  $r =$  \_\_\_\_\_ cm

Run	From Previous Activities			Calculate
	Experimental Acceleration $a$	Net Force $F_{\text{net assumed}}$ $(m_1 - m_2)g$	Total Mass $M_{\text{total assumed}}$ $(m_1 + m_2)$	$F_{\text{net assumed}} - M_{\text{total assumed}} a$
1				
2				
3				
4				
5				
6				
7				
8				

[m/s<sup>2</sup>]
[N]
[kg]
[N]

**From the plot:**

Slope,  $m' =$  \_\_\_\_\_ kg

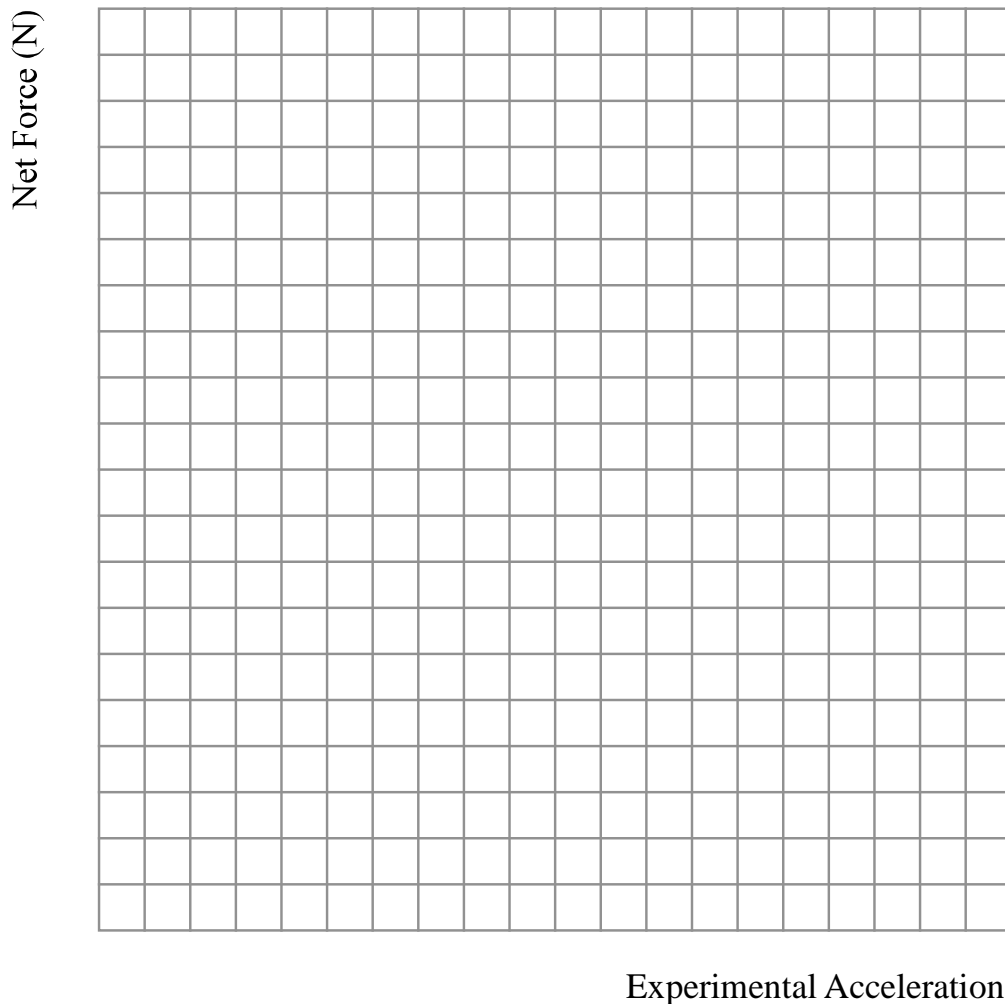
y-intercept,  $f' =$  \_\_\_\_\_ N

**Calculation:**

Rotational Inertia of the pulley,  $I =$  \_\_\_\_\_

**QUESTIONS PROCEDURE A: Total Mass Constant**

1. Look at the data: as the net force increased, what happened to the acceleration? Did it increase, decrease or stay constant?
2. Did a change in the net force produce a change in acceleration by the same factor? Do your results agree with Newton's 2<sup>nd</sup> Law?
3. Use this grid to make a plot of Net Force vs. Experimental Acceleration and draw the best fitting line.



4. Calculate the slope of the best-fitting line. What does the slope of the best-fit line represent? (Hint: what are the units of the slope?)

### **QUESTIONS PROCEDURE B: Net Force Constant**

1. Look at the data: as the total mass increased, what happened to the acceleration? Did it increase, decrease or stay constant?
  
2. Did a change in the total mass produce a change in acceleration by the same factor? Do your results agree with Newton's 2<sup>nd</sup> Law?

### **QUESTIONS PROCEDURE C: The Rotational Inertia of the Pulleys**

1. The motion and mass of the string that moves the system was never considered in any part of the theoretical analysis. Looking at your results, is it reasonable to ignore the mass of the string as part of the "total mass of the system"? Discuss.

2. Looking back at the ideal case: was it safe to assume that the system is essentially frictionless?
  
3. What has a larger impact on the percent differences found in procedures A & B, the small excess mass or the small amount of friction? Discuss based on your results.
  
4. Assuming frictional forces only act to oppose the linear motion of the masses, and that each mass receives the same amount of friction  $f$ , prove that  $f' = 2f$ .