Atwood's Machine: Applying Newton's Second Law (approximately 2 hr.) (10/27/15)

Introduction

A physical "law" is a statement of one of the fundamental theoretical principles that underlie our understanding of how the physical world works. Newton's First and Second Laws describe the relationship between force, mass, and acceleration. The net (total) force on an object, which is the vector sum of all applied forces, is equal to the total mass times the acceleration of the object. In this lab you will use a simple system of pulleys, strings and weights to investigate this relationship.

Equipment

- low friction pulley & clamp
- 0.9m rod w bench edge clamp
- pads to protect floor
- string
- level
- ruler
- 2 keyhole mass hangers
- 2 sets of keyhole masses
- grid paper
- 2-meter stick
- stopwatch

Note: String should be at least 130 cm. SAFETY NOTE: Mount rod & pulley below eye level.

Before the Lab

Read the sections in your text describing Newton's Laws. You should learn how to draw a force diagram (a.k.a. a "free-body" diagram) and determine the vector sum of all forces acting *on* an object.

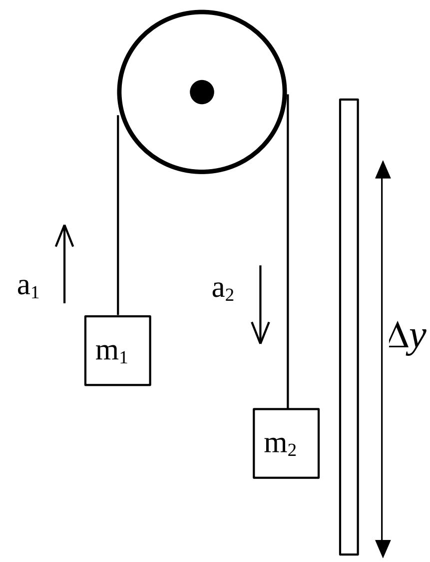

Figure 1: Mass and pulley system

Theory

When two masses are suspended by a string over a pulley (Figure 1) each feels a downward force due to its weight (W=mg) and an upward force due to the tension (T) in the string (Figure 2). If these two forces are equal, then the net force on the mass is zero and, as Newton's first law tells us, there will be *no acceleration* of the mass

If one mass is heavier than the other, then the masses may accelerate – one moving upward and the other down. Newton's second law tells us that in this case there must be a *net force* on each mass:

$$\vec{F}_{net} = m\vec{a}$$

If the string and pulley are considered to have negligible mass, then we can neglect the force needed to accelerate the string and cause the pulley to rotate. In this case the tension in the string is equal on both ends. The forces on the two masses are then:

$$F_1 = T - m_1 g = m_1 a \text{ and } F_2 = T - m_2 g = -m_2 a$$

where we have used the fact that the acceleration of each mass is the same (since they are connected by the string) and assumed that mass 1 is accelerating upward and mass 2 is accelerating downward. Algebraically we can solve this to show that

$$m_2 g - m_1 g = (m_1 + m_2) a$$

In effect the tension pulling in opposite directions cancels, so the net force becomes $F_{net} = T - m_1 g + m_2 g - T = m_2 g - m_1 g$ while the total mass being accelerated is $m_1 + m_2$. Thus $a = \frac{m_2 - m_1}{m_1 + m_2} g$ and if

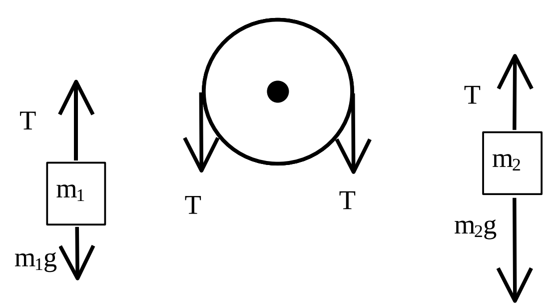

Figure 2: Force diagrams- draw each object separately

m_2 is greater than m_1 it will indeed accelerate downward. If m_1 is greater, the acceleration is negative meaning that each mass accelerates in the direction opposite to what was originally assumed.

Friction and Rotational Inertia

The real pulley and mass arrangement is not as simple as that described above. Some factors that will affect the acceleration are 1) friction, which has not been included in the net force, 2) the mass of the string, and 3) the *rotational inertia* of the pulley. If we add the force of friction, *f*, to the net force, it becomes $F_{net} = m_2 g - m_1 g - f$. The fact that the pulley is not massless means that it does require a net "torque" (a "turning" force) to make it rotate – this is supplied by the tension in the string. The *rotational inertia* of the pulley then adds an *equivalent mass* to the total mass being accelerated, so that $a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$ where the equivalent mass for the pulley is approximately $m_{equiv} = \frac{1}{2} M_{pulley}$. (Note: The mass of a solid disc pulley is approximately 100 g.)

(If you are using a red, spoked pulley, m_{equiv} is approximately 30 g.)

Determining the acceleration with a stopwatch and meter stick

The acceleration may be determined by measuring the time, *t*, required for the mass to fall from rest a specific distance Δy. If *a* is constant then we know that $\Delta y = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2$.

Procedure

You will be investigating the relationship between mass and acceleration by varying both m_1 and m_2 and observing *a*. We have shown from Newton's Second Law that

$$a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}.$$

Thus it seems worthwhile to try to systematically investigate the relationship between mass, force and acceleration by 1) keeping the net force constant by holding $(m_2 - m_1)g$ constant while varying $(m_2 + m_1)$ and 2) keeping the total mass, $(m_2 + m_1)$, constant while varying the net force by changing $(m_2 - m_1)g$

Vary the total mass (keep the net force constant)

Set up the apparatus as shown in the figure. Before assembling it record the weights of the mass hangers, the masses you will use, the pulley and the string. Use a piece of string long enough that you can observe the mass falling at least 1 meter. Use pad to protect the floor from impacts.

Begin with the two masses m_1 and m_2 each equal to 50 grams (this may be the mass of the hangers alone). When the two masses are equal the net force is zero. In the absence of friction, if the masses are set in motion they should continue to move at constant speed (zero acceleration). Hang the masses on the pulley and if necessary tap one side to set it in motion. Do you observe any significant friction? Record your observations.

Add 10 grams to m_2 but leave m_1 fixed. Start with m_2 near the pulley and measure the distance from the bottom of m_2 to the floor. This is Δy. Then release m_2 and record the time, *t*, which it takes to hit the ground. Make a few trial runs for practice and then record three independent measurements of *t*.

Add 100 grams to each side. This keeps the weight difference constant while increasing the total mass. Re-measure Δy (in case the string stretches) and record three independent measurements of time *t*.

Repeat three more times by adding additional increments of 100 grams to each side for a total of four trials with the 10-gram mass difference. Always re-measure Δy.

Now repeat for a total of 4 more trials using a 20-gram mass difference. (Note: Time permitting.)

Vary the net force (keep the total mass constant)

Begin with a total of 260 grams for *each* mass. For the ascending mass, m_1 use a combination of (50 + 200 + 5 + 2 + 2 + 1) grams. For the descending mass, m_2, use (50 + 200 + 10) grams. (Question: Should the mass of the mass hanger be included here? Why or why not?)

Transfer 2 grams from m_1 to m_2. Record the mass of each (ex. 258 grams and 262 grams) then measure Δy and make at least three good trials to record times and average.

Repeat by transferring additional mass from m_1 to m_2 in 1 gram increments:

Transfer an *additional* 1 g from m_1 to m_2 (i.e. 257 g and 263 g).
Transfer an *additional* 2 g from m_1 to m_2 (i.e. 256 g and 264 g, etc.)
Transfer an *additional* 3 g from m_1 to m_2.
Transfer an *additional* 4 g from m_1 to m_2.

Repeat, starting with a total of 360 grams for *each mass*. (Note: Time permitting.)

Analysis

1) For each trial, calculate the experimentally determined acceleration from the distance and time measurements. Show how the calculation is performed.
2) From the masses and mass differences calculate the theoretical acceleration. Include the effect of the equivalent mass of the pulley but neglect the effect of friction.
3) Calculate the percent difference between the theoretical and experimental values. (Percent difference is equal to $((Exp.–Theo.)/Theo.)\times 100\%$.)
4) For the data with <u>constant total mass</u>, the predicted relationship between acceleration and mass difference : $a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$ can be represented on a graph of acceleration ($a_{Exp.}$) versus mass difference (m_1 - m_2). A graph of y versus x is a straight line if it fits the formula "$y(x)=mx+b$", where "m" is the slope and "b" is the intercept. Rewrite the above equation for a in this form. What is the slope of the graph of a_{Exp} versus (m_2 - m_1)? What is the intercept. <u>Answer these questions in your report</u>.
5) Using a computer graphing program (EXCEL), make graphs of ($a_{Exp.}$) versus (m_2 - m_1) for each of your two data sets. Use a least squares fitting procedure (linear trend line) to find the slope and intercept of your graph. <u>Determine the units of the slope and intercept</u>.
6) From the slope and intercept, determine the gravitational acceleration, g and the force of friction, f. Make sure your units come out in m/sec^2 for acceleration and in Newtons for force.

Questions

Be sure that your report documents your calculations and explains how the value of g and the magnitude of the frictional force f can be determined from the graph.

How does the fact that the string has mass affect your experiment? Do you consider this an important effect?

Compare the magnitude of the force of friction to the other forces present in this experiment. Is friction a significant factor?

When the net force (due to mass difference) remains constant but the total mass increases, what happens to the acceleration?

When the total mass remains constant but the net force increases, what happens to the acceleration?

Explain specifically whether the answers to the previous two questions reflect Newton’s Second Law.

Vary the total mass (keep the net force constant): 10 gram mass difference

Mass of pulley ____________________ m_{equiv} for pulley ____________________

Mass of string ____________________ Mass of hangers ____________________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

Calculations (show and explain work)

Vary the total mass (keep the net force constant): 20 gram mass difference

Mass of pulley ____________________ m_{equiv} for pulley ________________________

Mass of string ____________________ Mass of hangers ________________________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

Calculations (<u>show and explain work</u>)

Vary the net force (keep the total mass constant at 260 grams)

Mass of pulley ____________________ m_{equiv} for pulley ________________________

Mass of string ____________________ Mass of hangers ________________________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

From graph:

Slope ________________ units

g from graph ________________ units

[ ] theoretical g units

Intercept ________________ units

f from graph ________________ units

Calculations (show and explain work)

Vary the net force (keep the total mass constant at 360 grams)

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, *t*	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

From graph:

Slope ____________ units   *g* from graph ____________ units   [   ] theoretical g units

Intercept ____________ units   *f* from graph ____________ units

Calculations (show work)