Conservation of Energy in a Pendulum (approx. 2 h) (6/6/2019)

Equipment

- pendulum clamp & string
- steel sphere with hook
- motion sensor
- Capstone software
- PASCO interface
- meter stick
- masking tape
- balance
- long, rigid rod (1.3m)

Introduction

The Work-Energy theorem tells us that changes in *kinetic energy*,

$$K = \tfrac{1}{2} mv^2,$$

are caused by *work* done by forces. A special case pertains when work is done by a *conservative* force. A conservative force is one for which the work done when traveling between any two points is independent of the path taken between the initial and final point. An example of this is the force of gravity (weight): work by gravity is always equal to *-mg* times the change in height, Δh. The amount of work done does not depend on how the object in question gets from one height to another, only on the final change in height. Furthermore, if the object returns to its initial height all of the energy lost to work against gravity can be recovered.

We can think of the negative work done against gravity as a *potential energy* that is being stored up and which can be recovered and converted back to kinetic energy by allowing gravity to do positive work. We define the change in potential energy, *U*, as

U= - work by a conservative force.

If there are no non-conservative forces (such as friction) the total *mechanical energy, E,* defined as

$$E=K+U,$$

is a constant. We say that the total mechanical energy is *conserved.*

For the gravitational force, the potential energy is given by:

$$U_{gravity} = mg\, \Delta h$$

where *mg* is the weight (force due to gravity) and Δh is the change in height. Note that potential energy can be negative: we define it as zero at some reference point, for example when we define the floor as zero height. Below this point the potential energy is negative, above this point the potential energy is positive.

In this lab we will investigate conservation of energy for a swinging pendulum. The experimental arrangement is shown below. A motion sensor is used to determine the position of the bob and calculate velocity. From the recorded position and velocity you will use a spreadsheet to calculate kinetic and potential energy: $K = \tfrac{1}{2} mv^2$, and $U=mg\, \Delta h$.

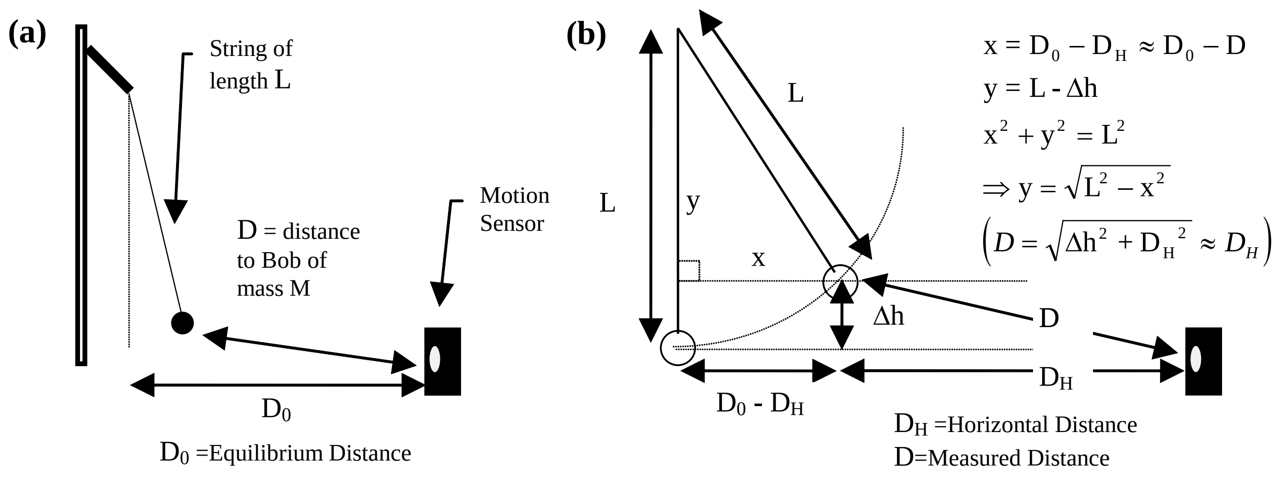

Figure 1: Experimental Set-up for recording the motion of a pendulum

Procedure

SAFETY NOTE: Adjust the pendulum clamp to be safely above eye and head level on the rod.

Aligning the pendulum and motion detector:

Measure and record the mass of the pendulum bob, m (in kilograms). Now move the detector very close to the bob, adjust the string so that the center of the bob is slightly below the center of the motion detector. Measure and record the length of the string, L. Move the detector away from the bob to perform the next observations.

Observations & Predictions:

Pull the pendulum away from the equilibrium hanging position by 10-20 cm and release it.

1. Describe below how the velocity and the height of the bob change as it moves.

2. Where is Potential Energy (PE) a maximum? Where is it a minimum?

3. Where is Kinetic Energy (KE) a maximum? Where is it a minimum?

4. How do you think the sum of KE and PE should change as the pendulum moves?
   Does this match your observations? If not, take another look.

Set up the motion sensor

Set the range switch on the motion sensor to the standard (STD) setting.

Place the motion sensor about 50 cm away from the bob. Make sure it's aimed at the bob and the sensor is aimed parallel to the table (i.e. level).

Start the Capstone program and display two graphs: position versus time and velocity versus time.

After you have set-up your experiment, measure and record the distance, D_0, from the bob to the sensor when the bob is stationary and hanging straight down. This is the equilibrium distance.

FOR BEST RESULTS: Be very accurate in this measurement. **If you move the pendulum clamp or the sensor, you will have to make this measurement again.**

Verify the equilibrium distance

With your meter stick you have measured the equilibrium distance, D_0, which is the distance from the detector to the bob at rest. You can measure this distance even more accurately by starting the program and recording a distance versus time graph for about 1 min. Using the Statistics function to determine the average this data should enable you to estimate the distance D_0 with a precision of 4 significant digits.

Record the motion of the pendulum

While carefully making sure the motion sensor does not move while you are doing your experiments (Use masking tape as a marker.), pull the pendulum bob back a distance of 5 cm or less and release it. (Small amplitude gives better results.) Start the program and the motion sensor will record the distance D as a function of time (see Figure 1). The program also takes the derivative of D, and displays the velocity, v, as a function of time.

For small amplitudes the measured distance D is approximately equal to the horizontal distance D_H, which is the quantity we use in the analysis. FOR SMALL AMPLITUDES WE TAKE $D \approx D_H$.

Analysis

Now we use the geometric relationships shown in Figure 1 (b) to calculate the pendulum's change in height, Δh, as a function of the distance, D, measured by the motion detector.

- When the pendulum is swinging we see: $y + \Delta h = L$
- From the triangle of x, y and L, we apply Pythagoras' theorem: $x^2 + y^2 = L^2$
- D_0 is the distance from the bob to the motion detector when the bob is at rest: $x = D_0 - D$.

Combine these three equations and derive an expression for the height Δh as a function of D (i.e. $\Delta h(D) = \ldots\ldots$). Show how you derive your formula.

Using the Data Launch Program to calculate KE and PE traces.

Now you will create two more graphs, one for the kinetic and one for the potential energy.

You will use the “Calculate” feature (tab at top of screen) to define new data series for kinetic energy and potential energy. For each velocity measurement in your selected data set the program will compute KE and will display the series of calculated KE’s as a graph. For each measurement of position D in your chosen data set, the program will calculate and display the PE in a separate graph. In order for this to work properly you must define KE and PE in the program based on fixed constants and the velocity or position measurements. For example, you will define KE for the program as .5*m*v^2. You will define m in this expression as the mass (in SI units) that you measured for the bob (a constant). You will define v in the formula by associating it with the chosen data set of velocity measurements. The program will then create a data set and graph of KE, one value for each measurement of v in the referenced data set. Similarly, you will define PE, but the definition is more complicated. For this, all of the parameters below must be defined:

Parameters needed for calculations:

Mass of Pendulum Bob, m	m =
Length of Pendulum, L	L =
Equilibrium Distance, D_0, (preferably from motion sensor ave.):	D_0 =
Formula for converting D_0-D to change in height Δh (gives Δh in terms of L, D_0, and D)	Δh =
Formula to calculate Kinetic Energy in Joules from velocity, v. (check the units!)	KE =
Formula to calculate the Potential Energy in Joules from distance D. (check the units!)	PE =

Before creating your derived data sets of KE and PE, check with your instructor that your formulas are correct and in the form needed by the computer program.

Note that the program only understands multiplication and exponentiation when the specific symbols are used as indicated below. Factors just typed consecutively are not understood as factors in multiplication. Also, it will be necessary to define mass (m) individually for each derived data set (formula).

Notes for formulae input:

Multiply numbers with an asterisk: *.
Raise numbers to a power using the symbol: ^. (For example x^2 is written as x^2.)
Use plenty of parentheses to factor.
A square root is given by SQRT(x) or by (x)^0.5.

<u>Kinetic Energy:</u> Determine the ***formula*** that will allow you to calculate the kinetic energy, in joules, from velocity, v, as measured by the detector. Click on the 'Calculator' tab on the tools palette on the left side of the screen in Capstone. After opening the Calculator tab, press 'New' and then change the name of the calculation to KE for kinetic energy and enter the units of Joules. Double click in the equation box to the right of the "KE=" to enter the equation (i.e., $1/2mv^2$). When entering the equation, type 0.5*(the numeric value of the mass in kg.)*[Velocity]^2. The program will perform a KE calculation for each velocity data point. NOTE: When you enter the equation you will select the required quanitity (i.e., velocity) from a drop down tab that appears after typing the bracket "[" button. Press the enter key after defining the equation and check your KE data by displaying a graph.

<u>Potential Energy:</u> Determine the formula that will allow you to calculate the potential energy, in joules, from distance, D, as measured by the detector. Then follow the same procedure as for the kinetic energy. In order to define a new formula, click on 'New,' Label the calculation PE, change the units to Joules and enter the equation: PE = mgh

PE= (the numeric value of the mass in kg.)*9.8*(height).

Remember you are expressing "h," the height of the pendulum-bob in terms of L, D_0, and D. You may prefer to calculate "h" as a function of the other variables and perform a calculated column for before performing the PE Calculation. When entering an equation for 'h,' the height of the pendulum, you can select the distance (D) using the bracket (e.g., "[") key to access the distance data collected by the motion sensor. Finally, create a new calculated column, showing the total energy as a function of time.

TE = KE + PE

Discussion of Results:

Sketch, screen save, or print the graph, which shows both PE and KE.

Add a legend to the graph explaining what it is.

On your graph or another piece of paper write a verbal description and explanation of what is shown in the graph.

Discuss your charts of KE and PE by comparing them to the predictions you made

1.When KE is a maximum?What is PE then?
When PE is a maximum?What is KE then?

2.What does your graph reveal about the total energy, TE? Does it vary? How much?

3.Is there any evidence that the approximation of the height of the pendulum bob is causing you trouble?

4.Estimate the % error introduced into your calculation of E by neglecting the size of the bob.

5.Are there some signs that the equilibrium distance D_0
If so make corrections. is not chosen correctly?

6.Does the graph of the total energy show that the total energy is conserved?

How would frictional losses alter the graph?

For your report write a one-page cover sheet with an explanation of the theory of the experiment in your own words. Include data below. Print out chart from spreadsheet showing KE, PE and total E with explanation and discussion of the graph. Be sure to discuss how the graph relates to the motion of the pendulum. Answer all applicable questions.