# Rotational Inertia (approximately 2 hr) (11/23/15)

## *Introduction*

In the case of linear motion, a non-zero net force will result in linear acceleration in accordance with Newton's $2^{nd}$ Law, **F**=m**a**.   The moving object will posses kinetic energy, K= ½ mv$^2$. For rotational motion, a net unbalanced, non-zero torque, $\tau$, will result in angular acceleration, $\alpha$, according to the equation $\tau$=I$\alpha$, where I is the rotational inertia of the object, and the rotating object will have rotational kinetic energy, K= ½ I $\omega^2$. The rotational inertia (or moment of inertia) depends not only on the mass but also on how the mass is distributed about the axis of rotation.

## *Equipment*

- rotational platform
- solid disc
- cylindrical shell
- 2 solid cylinders
- rod and screws

- mass set
- 50 g mass hanger (angled)
- 5 g mass hanger
- level

- bench-edge pulley
- meter stick
- caliper
- stop watch

For the class as a whole: balance, screwdrivers (both types)
Note:  string length approx. 1.3 m

## *Theory*

Objects having different rotational inertia will be placed on a rotational apparatus.  A bench-edge pulley will be aligned with the wound string coming tangentially off the round base of the rotational stand. (See Fig. 1)  A hanging mass produces tension in the string, which applies a torque to the rotational apparatus. We can measure rotational inertia by looking at either the angular acceleration of objects placed on the rotating platform or by looking at the increase in rotational kinetic energy.   For the hanging mass, Newton's Law predicts:

$F = mg - T = ma$

while for the rotating platform the torque is given by:

$\tau = r_\perp F = r_\perp T = I\alpha$,

where $r_\perp$ is the radius of the part of the platform which the



**Figure 1: Rotational Apparatus**

string is wrapped around. If the string unwinds without slipping, then we also have the relationships:

$$\alpha = a / r_\perp \qquad \omega = v / r_\perp \qquad \theta = s / r_\perp$$

where *a* and v are the acceleration and velocity of the string as it unwinds, $\alpha, \omega,$ and $\theta$ are the angular acceleration, angular velocity and angle in radians of the platform as it turns, *s* is the arc length of the circle (which is equal to the length of string unwound) and $r_\perp$ is the radius about which the string is wound.

In this lab we will relate the linear motion of the falling mass to the angular motion of the platform. This requires the equations for constant acceleration:

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2 \qquad v = v_0 + at \qquad v^2 = v_0{}^2 + 2a\Delta x \quad \text{and} \quad g = 9.8 \frac{m}{s^2}$$

Alternatively if we apply Conservation of Energy then the gravitational potential energy of the mass (which it possesses due to its initial height h) is converted to kinetic energy of both the falling mass and the rotating platform. Before the mass falls (if the system starts from rest) K=0 and all the energy is potential E= mg h. After the mass falls from a height h to height of zero, all of the energy
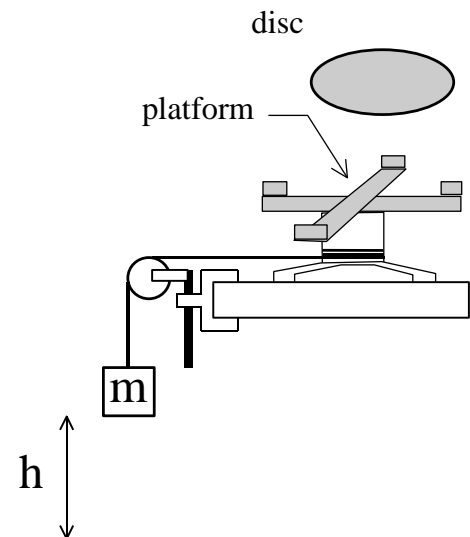
is kinetic and E= ½ mv² + ½ I ω². If there is no friction then E is a constant and mgh = ½ mv² +½Iω² .

***Procedure*** (**Note:** *Always guide string & keep tension on it or it will get tangled in bearings!*)

Make sure the string is wound around the circular base and does not go beneath it and become wrapped around the small axial post. If the string becomes caught in the axial post of the rotation platform then turn the platform upside down to take its weight off the string --- the string can then be more easily unwound. (If this doesn't work you may need to disassemble your platform.)

Level the rotational platform and adjust the bench-edge pulley so it is aligned with the string and the string pulls horizontally on the platform as the mass falls vertically.

Using a vernier caliper, measure the diameter of the part of the apparatus around which the string is wrapped. (See Appendix 4 in online lab manual if you don't know how to read a vernier.) Divide the diameter by 2 to get the radius. Measure the height that the mass will fall to the floor. Record these as $r_\perp$ = $R_{string}$, and h. (Note: The effective radius at which the string pulls actually changes as the string unwinds from the shaft, due to the overlapping of the string. Can you devise a way to get a better estimate of the average effective radius of the shaft as the mass descends?)

With the string and pulley in place, use the 5g mass hanger to suspend just enough mass, $M_0$, to produce slow uniform motion (constant speed) once you have started it moving. This will be just enough to counterbalance any frictional forces. In later parts of the experiment this amount of mass will not be counted as part of the hanging mass that produces a net force. Record the weight of this mass in the table as the "Zero Acceleration" weight, $W_0$=$M_0$g.

Now replace the 5 g mass hanger with a 50 g mass hanger (angled), and place enough mass on it to exceed the zero acceleration mass by about 70 g (or enough to produce significant acceleration). Measure the time that it takes the mass to fall from rest until it hits the floor. *For good results the time of fall should be at least 4 seconds.* Make three trials making sure to use the same initial height each time and average your results.

Assume constant acceleration and calculate the acceleration, a, of the mass from the known initial height, h, initial velocity (i.e. zero) and time of fall.

Calculate the angular acceleration, $\alpha$ , of the rotating platform assuming the string did not slip. Once you know the acceleration of the mass you know the net force on the mass.

Draw a force diagram of the falling mass and calculate the tension, T, in the string (do not include the "Zero Acceleration" weight $W_0$ in this calculation, <u>do</u> include the added weight, W).

Draw a force diagram for the rotating platform (using an overhead view) and calculate the torque applied to the platform by the tension, T, in the string.

Find the rotational inertia of the platform, $I_0$.

## Determining Rotational Inertia of an Object Added to the Platform

Now you will add various objects onto the platform and determine their rotational inertia. If the new object has rotational inertia I then the total rotational inertia of both the platform and the object on top of it is $I_{total} = I_0 + I$. You will measure $I_{total}$ then subtract $I_0$ (as determined above) to find the rotational inertia of the additional object, I.

Use the following procedure to determine the rotational inertia for the solid disc, the hollow ring and the bar.

Make a sketch of your object and record all of its dimensions and mass. Place the new object on the platform, making sure to align its center with the axis of rotation. (Screws will be needed to attach the bar.)

Repeat the measurement of the zero acceleration weight, $W_0$, since the forces of friction will change with changes in weight.

Use the procedure outlined above to determine $I_{total}$. Subtract your previously obtained value for $I_0$ to find the rotational inertia of the object you added.

Calculate the percent difference between the rotational inertia you measure and the theoretical value (See Table below).

**Rotational Inertia of various objects about an axis through the center of mass**

| Solid Disc or Cylinder, radius R | Hollow Disc or Cylinder, inner radius $R_2$, outer radius $R_1$ | Bar, length A and width B |
|---|---|---|
| $I = \frac{1}{2} MR^2$ | $I = \frac{1}{2} M (R_1^2 + R_2^2)$ | $I = \frac{1}{12} M (A^2 + B^2)$ |

## *Additional Exercises*

### Rotational Inertia About an Axis Parallel to the Center of Mass

The parallel axis theorem states that the rotational inertia of an object about any axis is equal to the rotational inertia about a parallel axis through the center of mass plus the mass times the distance, d, between the axes squared: $I = I_{CM} + Md^2$. In this exercise we will consider the rotational inertia $I_0$ of the "platform" to be the rotational inertia of the platform plus the rotational inertia of the bar, as previously determined (i.e. $I_{total}$ in the table on p. 7).

Measure the diameter of a solid cylinder, divide by 2 to get its radius, R, and calculate its rotational inertia about an axis thru its center of mass using $I_{CM} = \frac{1}{2} MR^2$. Assume this to be the correct value.

Measure the distance, d, from the center (i.e. midpoint) of the bar to the center of the second hole from the midpoint and fasten the solid cylinder to the bar there with a thumbscrew. (We will neglect the inertia of the thumbscrew.)

Attach the bar/cylinder assembly to the rotational platform using 2 thumb screws.

Determine the total rotational inertia of the entire rotating assembly and then calculate the rotational inertia of the solid cylinder about the axis of rotation of the platform. Think this thru and carefully explain your calculations and why they are valid.

Make a sketch of the arrangement, label all dimensions and confirm the parallel axis theorem.

### Conservation of Energy

For one set of data show that energy is approximately conserved as the mass falls (decreasing its potential energy) while the platform and added mass accelerate rotationally (increasing their kinetic energy).

## *Questions:*

Calculate the ratio of the mass of the disc to the mass of the ring. Calculate the ration of the rotational inertia of the disc to the rotational inertia of the ring. Compare these two ratios. If they are not equal then rotational inertia must not depend on mass only. What other factors effect rotational inertia? Discuss.

Compare the moments of inertia of the different objects. Discuss how they differ in their physical dimensions, masses, and how the mass is distributed. Do the measured values for rotational inertia vary as you would predict?

How do your moments of inertia compare to the theoretical predictions? Discuss possible sources of error or uncertainty. (Do your objects match their theoretical descriptions? How big of an error is any difference?)

Why doesn't rotational inertia depend on the height (thickness) of the objects?

Compare the rotational inertia for rotating on-center and off-center. Does it vary as you would expect?

**Rotational Inertia of Platform ("Cross")**

| Sketch: | Initial Height: |
|---|---|
| | Radius $r_\perp$ =R$_{string}$: |
| | Mass needed for "Zero Acceleration": <br> M$_0$= <br> Weight W$_0$ = |
| Mass added to M$_0$: <br><br> Weight added, W: | Times measured: <br> 1) <br> 2) <br> 3) <br> Average: |

Force Diagrams and Calculations. Work out on scratch paper, then write clearly, show all steps and label and discuss each.

Results of Calculations:

| Accel of mass, a | Angular accel. $\alpha$ | Tension in string, T | Lever-arm for torque | Torque, $\tau$ | Rotational Inertia, I$_0$ |
|---|---|---|---|---|---|
| | | | | | |

**Rotational Inertia of Solid Disc**
Description of this Object and Procedure Used:

| Sketch: | Initial Height: |
| --- | --- |
| | Radius $r_\perp$ =R$_{string}$: |
| | Mass needed for "Zero Acceleration": M$_0$= <br> Weight W$_0$ = |
| Mass:　　　　Dimensions: | |
| Mass added to M$_0$: <br><br> Weight added, W: | Times measured: <br> 1) <br> 2) <br> 3) <br> Average: |

Results of Calculations:

| Accel of mass, a | Angular accel. $\alpha$ | Tension in string, T | Lever-arm for torque | Torque, $\tau$ | Rotational Inertia, I$_{total}$ | I$_0$ | I for this object |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | | |
| Experimental Value for I (from above) | | | Theoretical Value for I | | | % Diff: | |

Discussion and Calculations. Work out on scratch paper then <u>write neatly and label or discuss all parts</u>. Include extra paper or write on back if needed.

**Rotational Inertia of Hollow Cylinder**
Description of this Object and Procedure Used:

| Sketch: | Initial Height: |
|---|---|
| | Radius $r_\perp = R_{string}$: |
| | Mass needed for "Zero Acceleration": $M_0=$ <br> Weight $W_0 =$ |
| **Mass:**      **Dimensions:** | |
| Mass added to $M_0$: <br><br> Weight added, W: | Times measured: <br> 1) <br> 2) <br> 3) <br> Average: |

Results of Calculations:

| Accel of mass, a | Angular accel. $\alpha$ | Tension in string, T | Lever-arm for torque | Torque, $\tau$ | Rotational Inertia, $I_{total}$ | $I_0$ | I for this object |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| Experimental Value for I (from above) | | | Theoretical Value for I | | | % Diff: | |

Discussion and Calculations. Work out on scratch paper then <u>write neatly and label or discuss all parts</u>. Include extra paper or write on back if needed.

**Rotational Inertia of Rectangular Bar**
Description of this Object and Procedure Used:

| Sketch: | Initial Height: |
| --- | --- |
| | Radius $r_\perp = R_{string}$: |
| | Mass needed for "Zero Acceleration": $M_0 =$ Weight $W_0 =$ |
| Mass:          Dimensions: | |
| Mass added to $M_0$: Weight added, W: | Times measured: 1) 2) 3) Average: |

Results of Calculations:

| Accel of mass, a | Angular accel. $\alpha$ | Tension in string, T | Lever-arm for torque | Torque, $\tau$ | Rotational Inertia, $I_{total}$ | $I_0$ | I for this object |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | | |

| Experimental Value for I (from above) | Theoretical Value for I | % Diff: |
| --- | --- | --- |

Discussion and Calculations. Work out on scratch paper then <u>write neatly and label or discuss all parts</u>. Include extra paper or write on back if needed.

**Rotational Inertia Off-Center Solid Cylinder about a Parallel Axis**
Description of this Object and Procedure Used:

NOTE:  In this exercise the "platform" is the original platform (cross) plus the rectangular bar.

| Sketch: | Initial Height: |
|---|---|
| | Radius $r_\perp = R_{string}$: |
| | Mass needed for "Zero Acceleration": $M_0 =$ <br> Weight $W_0 =$ |
| Mass:          Dimensions: | |
| Mass added to $M_0$: <br><br> Weight added, W: | Times measured: <br> 1) <br> 2) <br> 3) <br> Average: |

Results of Calculations:

| Accel of mass, a | Angular accel. $\alpha$ | Tension in string, T | Lever-arm for torque | Torque, $\tau$ | Rotational Inertia, $I_{total}$ | $I_0$ (incl. bar) | I for this object |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

| Experimental Value for I (from above) | Theoretical Value for I = $I_{CM} + Md^2$ <br> ($I_{CM} = \frac{1}{2} MR^2$) | % Diff: |
|---|---|---|

Discussion and Calculations. Work out on scratch paper then write neatly and label or discuss all parts. Include extra paper or write on back if needed.

# ADDENDUM TO ROTATIONAL INERTIA LAB
## Alternative Method for Calculating Moment of Inertia Using Energy Conservation
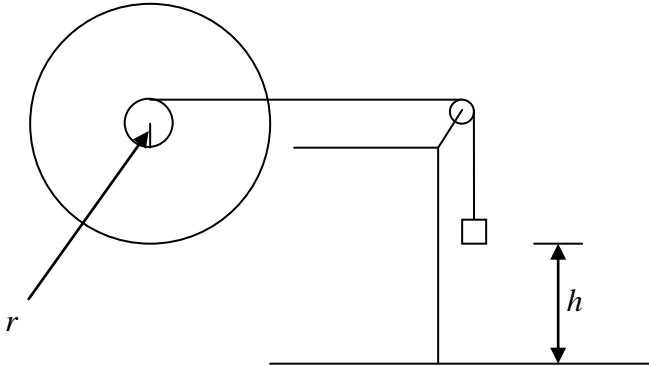


I.    Energy lost as friction:
   - a. Find a mass $m_0$ such that the speed of that mass going down is constant.
       - i. Start with the system at rest.
       - ii. Apply a small force to start the system moving at low speed. If the mass descends at constant speed, then $m_0 g$ equals the force of kinetic friction.
       - iii. If the system does not continue to move add some mass.
   - b. Mechanical energy lost due to friction is $\Delta W_f = \vec{F}_f \cdot \Delta \vec{x} = m_0 g h$. ( $F_f = m_0 g$ )

II.    Angular speed of disk $\omega$
   - a. The linear speed of the string $v$ is related to $\omega$ by $v = r\omega$
   - b. Thus, $\omega = v/r$. We need to measure $r$ carefully.

III.    How to measure $r$
   - a. Measure initial position of hanger.
   - b. Let the support rotate 4 revolutions exactly. Measure final position of hanger.
   - c. The distance the hanger moved ($D$) is related to the effective radius $r$: $D = 4(2\pi r)$
   - d. Thus, $r = D/(8\pi)$

IV.    How to measure the final velocity of hanging mass when it accelerates ( $m \approx 1{,}000$ gr )
   - a. Measure the distance the hanger falls to the floor $h$
   - b. Measure the time it takes $t$ to fall to the floor (starting from rest).
   - c. Using the equations of motion with constant acceleration you will easily find that $v = -2h/t$

V.    To measure $I$
   - a. Use energy conservation: $mgh = \dfrac{1}{2} I\omega^2 + \dfrac{1}{2} mv^2 + m_0 gh$
   - b. Solve for $I$ in this equation.
   - c. You need $I$ for the a) support, b) support+disk, c) support+ring
   - d. Since $I$ adds like a mass you can find $I$ for the disk and ring by subtracting the moment of inertia of the support

VI.    Compare with theory
   - a. $I_{Theory} = \dfrac{1}{2} M\left(R_1^2 + R_2^2\right)$
   - b. Find the percent difference: $\%\,diff = \dfrac{\left|I_{Theory} - I\right|}{I_{Theory}} \times 100$