Balancing Torques and the Center of Gravity (approx. 1.5h) (11/20/15)

Introduction

Just as force is required to accelerate a mass, a *torque* is required to produce angular acceleration. A torque is a force applied at a distance offset from some axis of rotation. This distance is often called the *lever arm* and the larger the lever arm is the more torque can be produced by the force. The torque, τ, is defined by

$$\tau = r_{\perp} \times F$$

where $r_{\perp}$ is the *lever arm:* the distance from the axis of rotation to the line of the applied force, as shown in Figure 1. For rotation in two dimensions, torques can cause counter-clockwise (positive by convention) rotation or clockwise (negative) rotation.

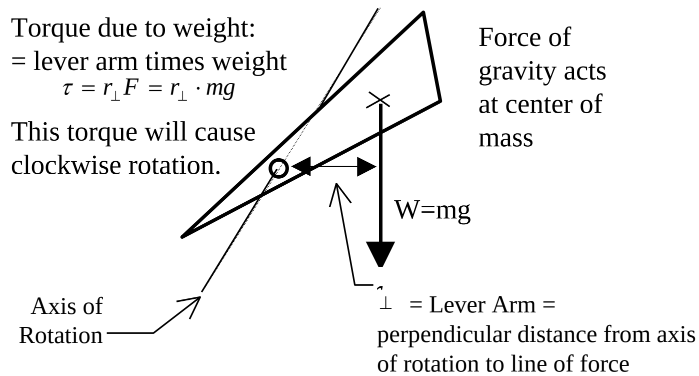

Figure 1: The weight of an object can cause a torque about an off-center axis

Equipment

- meter stick
- 4 meter-stick clamps, (3 with "hangers")
- 5g mass hangers (3)
- balance
- pivot stand
- unknown mass
- slotted mass set

Theory

In this lab you will study the special case of an object in *static equilibrium*: it is neither moving nor accelerating linearly, or rotationally. This means that there is no net force and no net torque on the object. Any forces and torques applied to the object must be balanced. For example any torque which would cause clockwise rotation must be balanced by a torque which would cause counter-clockwise rotation.

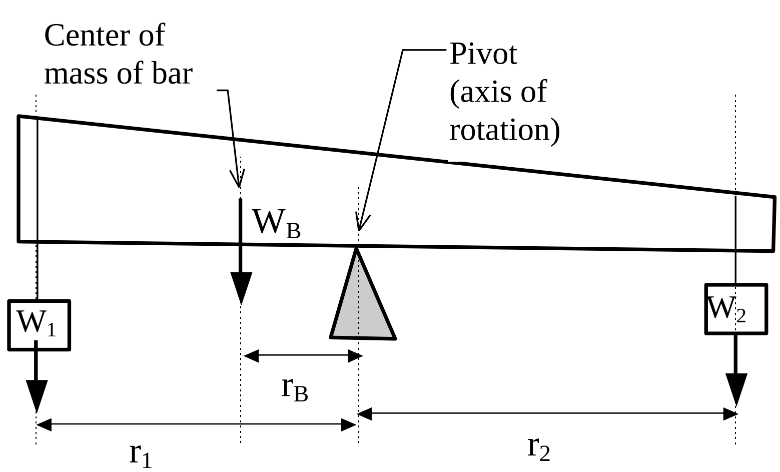

Figure 2: Objects balanced on a pivot: there is no net force and also no net torque. The weight acts at the center of mass.

One of the external forces on the object might be its weight. Although the force of gravity acts on every point throughout the object, the net effect is the same as if all the mass were concentrated at the *center of mass.* We say that the *weight acts at the center of mass,* which in the laboratory setting is also referred to as the *center of gravity.*

Figure 2 shows a bar balanced on a pivot. If the bar were to rotate then the pivot would be the axis of rotation. In order for it not to rotate the torques produced by all the forces must "balance out" (i.e. add up to zero). The weight, $W_B = m_B g$, of the bar itself acts at the center of mass of the bar. The lever arm for this force is shown as r_B so the torque it would produce is $\tau = r_B W_B$ which would tend to make the bar rotate counterclockwise. Similarly a hanging mass M_1 also produces a positive counterclockwise torque , $r_1 W_1$, while the second hanging mass, M_2, produces a negative, clockwise torque, $- r_2 W_2$. For the bar to be balanced the net torque must be zero:

$\tau_{total} = r_B W_B + r_1 W_1 - r_2 W_2 = 0$

or

$$\tau_{CCW} = - \tau_{CW} \qquad \text{so that} \qquad r_B W_B + r_1 W_1 = r_2 W_2$$

Getting a "feel" for torque:

Hold one end of the meter stick with one hand and slide a clamp with a hanger onto the other end and move it very close to your hand. Using a 5 g mass hanger, hang a 500 g mass from the clamp, holding the meter stick still and horizontal. Now, move the clamp and suspended mass farther and farther from your hand while keeping it still and level. Notice that this becomes increasingly difficult as the mass gets farther from your hand, even though the total mass (and therefore weight) of the system has not increased at all! What you feel is the increased torque your hand must exert on the meter stick to maintain equilibrium by counteracting the increased torque exerted by weight of the attached mass as its lever arm increases

Procedure

A meter stick balanced on a pivot will be used to investigate how balancing torques results in equilibrium.

A. Meter stick supported at center of gravity:

Determine the weight of your meter stick and the three clamps with hangers, which you will use to hang weights. Record this data. (See p.4, Table A.)

Adjust the position of the meter stick in the center clamp (the one without a hanger) until you can balance it on the pivot. In order for the meter stick (plus clamp) to have a position of *stable equilibrium,* their center of gravity must be directly beneath the pivot point, so the clamp should hang downward from its support projections. Balance the meter stick on the pivot by moving it inside the center clamp until you find the equilibrium position (i.e. meter stick level). The center of mass should now be directly beneath the pivot point. Tighten the clamp screw and record this position (on p. 4).

Two known, unequal weights

Slide the other two clamps onto each end of the meter stick with the hangers hanging down. Use two 5 g mass hangers to hang unequal masses from the clamp hangers. Adjust their positions until you achieve equilibrium (i.e. level meter stick). Record the total weights (W) and their positions on the meter stick (X) as determined by the locations of the projections on the clamps. Make a sketch of the arrangement and label the positions of the weights and the center of mass. (Which weights should be included in the total weight at each location?)

Calculate the lever arm for each weight. Indicate this distance on your sketch as well.

Calculate the net clockwise torque and the net counterclockwise torque. Find the percent difference between the two. How does this compare to the condition required for balance?

As you move the two weights how is the rotational equilibrium disturbed? Is there more than one way to position the two weights to achieve balance? Vary the positions of the weights until you have determined a general condition for balancing the weights. Record your observations and conclusions.

In the second data table record the weights and positions corresponding to your new equilibrium. Calculate the CCW and CW torques and their percent difference.

Three known weights (not all the same)

Repeat using three weights (at least two different) at three different positions. Find an equilibrium configuration. Sketch your configuration and record your weights and positions. Compute total clockwise and counterclockwise torques and their percent difference.

Now choose two different positions for the first two weights. Calculate the lever arm required for the third weight to balance these two. Record your prediction then experimentally find the actual position required. Calculate the percent difference between the predicted and measured lever arms for the third weight.

Unknown Mass:

Choose a lead weight to use as an unknown mass. With the meter stick balanced at its center of mass, hang the unknown mass on one side. On the other side place a known mass and adjust its position until you find balance. Describe the calculations needed to find the unknown mass and record your results. (Don't forget to include the effects of any clamps or mass hangers you use.)

Determine the weight of your unknown mass with the triple-beam balance and compare the result to your calculation.

B. Meter stick supported away from center of gravity:

With only one weight suspended from one end of the meter stick, loosen the center clamp and move the meter stick through it until the extended meter stick balances the attached weight (i.e. level).

Calculate the total torque about the pivot due to the hanging mass + clamp + 5 g hanger.

This torque is being balanced by the weight of the meter stick, which acts as if it were all concentrated at its center of mass! Pretend that the meter stick is massless. Where would you need to hang a second mass, equal to the mass of the meter stick, so that its lever arm would be sufficient to balance the total torque due to the attached masses? Calculate this lever arm (from the new position of the pivot). Compare its position to the previously determined position of the center of mass of the meter stick.

Questions

When two unequal masses are balanced on either side of the (balanced) meter stick, what is the relationship between the ratios of the masses and the ratios of their lever arms?

Is it possible that there is no net (total) force on an object (such as the meter stick) but a non-zero torque? Explain and give an example.

How does the triple-beam balance work?

A. Meter Stick Supported at Center of Gravity

Mass of stick:		
Mass of first clamp: Weight=	Mass of second clamp Weight=	Mass of third clamp Weight=
Position of Center of Mass of meter stick:		

Two Known Weights Data Table (first equilibrium arrangement of weights)

Sketch (labeled)	Weights, Position	Lever Arms:	Results:
	W_1= X_1 = W_2= X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Discussion of how the balance of the meter stick varies as the weights are moved:

Two Known Weights Data Table (second equilibrium arrangement of weights)

Sketch (labeled)	Weights, Position	Lever Arms:	Results:
	W_1= X_1 = W_2= X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Additional Discussion:

Three Known Weights Data Table (first equilibrium arrangement of weights)

<table>
<tr><td rowspan="3">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td>Lever Arms:</td><td>Results (include direction)</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =
W_3= X_3 =</td><td>r_1=
r_2=
r_3=</td><td>τ=
τ=
τ=</td></tr>
<tr><td>τ_{CCW}=</td><td colspan="2">τ_{CW}=</td><td>% Diff:</td></tr>
</table>

Three Known Weights Data Table (second equilibrium arrangement of weights)

<table>
<tr><td rowspan="4">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td colspan="2">Lever Arms:</td><td>Results:</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =</td><td colspan="2">r_1=
r_2=</td><td>τ=
τ=</td></tr>
<tr><td>W_3=</td><td colspan="4">Predicted position and lever arm:</td></tr>
<tr><td colspan="3">Measured position and lever arm:</td><td colspan="2">% Diff. (lever arms):</td></tr>
</table>

Discussion and Observations:

Unknown Mass

Sketch (labeled)	Weights, Position	Results:
	W_1= ??? X_1 = W_2= X_2 =	W_1 (calculated) W_1 (measured using balance) % Diff. =

Describe calculations needed to determine unknown mass (here or on additional page):

B. Meter Stick Supported Away from its Center of Gravity

Mass of stick (previously measured, Table A.)
Position of Center of Mass of meter stick (previously measured)
Position of pivot:

Sketch (labeled)	Weight, Position	Lever Arm: (from pivot)	Torque:
	W_1= X_1 =	r_1=	τ=
	Lever arm required for weight (equal to weight of meter stick) to balance attached weight:	Position required for weight equal to weight of meter stick to balance attached weight:	
Compare predicted position to center of mass:			

Calculations and Discussion: