Standing Waves on a String (approx. 1.5 hours) (2/21/11)

Introduction

A mechanical wave is a motion disturbance that propagates in some medium. In air, disturbances propagate in the direction of motion and are called *longitudinal waves*. Sound waves are longitudinal waves. In solids, it is also possible for disturbances to propagate perpendicular to the direction of motion. These are called *transverse waves*. In this lab you will study "standing" transverse waves on a string bounded at both ends.

Equipment:

Strings: (approx. 1 hr.)

• string vibrator with string	• bench-edge clamp & threaded rod	• mass set • 5g mass hanger	• AC power supply (adjustable amplitude)
• meter stick	• bench-edge pulley	• rod clamp	

For class as a whole: string sample; high-precision electronic balance; Phillips screw driver.

NOTE: Linear density of white pulley cord is about 3.84 x 10^{-4} kg/m.

Theory

Waves:

Common characteristics of waves are

Wavelength λ: the minimum distance for the pattern of the wave to be repeated.

Frequency f : the number of times per second the wave motion is repeated (measured in Hertz: 1Hz=1/s).

Period T: The time required to make one oscillation or for the wave to travel one wavelength.

Speed v: the velocity of propagation of a traveling wave: $v = \lambda \cdot f$

Amplitude A: The maximum size of the disturbance.

A *Node* in a wave is a position where the size of the disturbance is zero. An *anti-node* is the position of maximum disturbance.

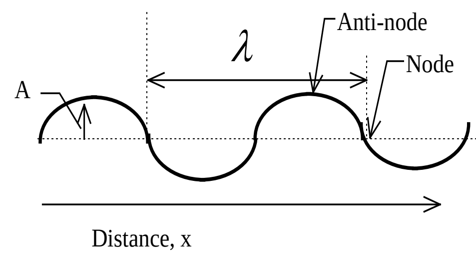

Figure 1: A standing wave oscillates in place at frequency, *f*. A traveling wave will propagate at speed, *v*. At any one point the disturbance oscillates with frequency, *f*, such that $v = \lambda \cdot f$

Waves on a string:

A wave on a string is an example of a *transverse* wave: the disturbance in the string (displacement from a straight line) is transverse (perpendicular) to the direction of the string. The wave may be *traveling* down the length of the string or be a *standing wave* which oscillates in place. Waves on a string have characteristic frequencies which depend on the linear density of the string, μ (i.e. mass per unit length), and the tension, τ, in the string. The speed of the wave in the string is given by: $v = \sqrt{\tau/\mu}$. Even though standing waves do not appear to travel along the string, their wavelength and frequency are still characterized by $\lambda \cdot f = \sqrt{\tau/\mu}$.

Resonance:

Resonance occurs when the driving frequency creating waves matches a natural frequency of the bounded mechanical medium in which the waves propagate. In a string with both ends fixed, standing waves can be supported only if there is an integer number of half wavelengths along the string ($L = n \cdot \lambda/2$): in this case the ends of the string are at nodes which remain fixed. The characteristic, or resonant frequencies of the string are then determined from the relationship: $\lambda \cdot f = \sqrt{\tau/\mu}$

Procedure

Mount a bench-edge pulley on one end of your lab bench and set up a vertical rod using a bench edge clamp located about 1.5 m from the pulley. Clamp the string vibrator rod onto the vertical rod and adjust its position such that the length of the (horizontal) string from the tip of the vibrator reed to the top of the pulley wheel is 1.33 m. (See Fig. 2.)

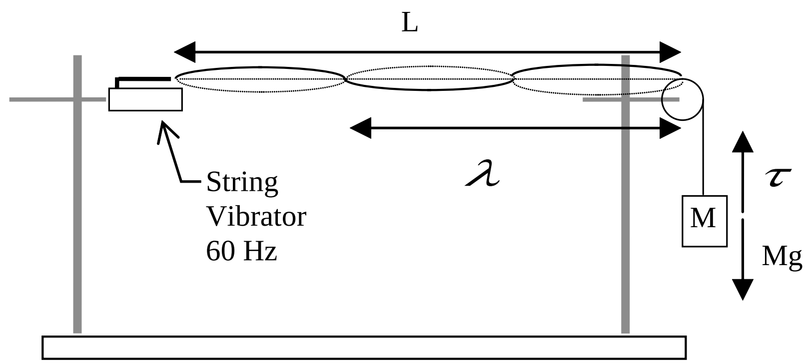

Figure 2: Standing Waves on Vibrating String

The vibrator uses an electromagnet which alternately attracts and repels a steel "reed". The magnet operates using AC line voltage, so its fundamental driving frequency is 60 Hz. However, there are always low amplitude "harmonics" presents in electromechanical systems. So, under certain circumstances, a harmonic of 60 Hz (i.e. 120 Hz, 180 Hz, … etc.) may excite a standing wave on your string. Discard such data unless otherwise instructed.
Your instructor may want you to measure and record the mass m and length L of a piece of string like the string on your vibrator and calculate its linear density in kg per meter. Otherwise, use *3.84 x 10⁻⁴ kg/m.*

Attach a 5 g mass hanger to the end of the string hanging over the pulley. This will apply a tension $\tau = Mg$ to the string, where M is the mass of the hanger. (The mass of the string may be considered negligible.) Record the length (L) of the string between its fixed ends (i.e. from the vibrator to the pulley). Plug the vibrator into the variable voltage 60 Hz power supply and turn the power on to full amplitude. While watching the string for the occurrence of standing waves, add mass to the hanger, increasing it 1 g at a time. Adjust the mass on the hanger so you can clearly see standing waves (i.e. nodes and anti-nodes) occurring. Try to produce a good stable standing wave pattern on the string by adding or subtracting mass to or from the mass hanger. (Hint: See if you can get 8 loops (i.e. anti-nodes) by placing about 10 g on your 5 g hanger.) If your reed is clanging noisily and your nodes and anti-nodes are unstable it may help to slowly decrease the voltage output of your power supply to get a quieter, more stable pattern. If you've gotten a good pattern with stable, clearly discernable nodes, measure L_n, the distance between the node on the top of the pulley and the node closest to the vibrating reed. (Although the end of the string attached to the reed cannot be exactly at a true node, since the reed is typically vibrating with amplitude of at least 1 or 2 mm, it may be "close enough" in this particular case to consider it to be a node point.) Turn your power supply off (to prevent over heating of the vibrator electromagnet) and record your data in the table on the last page.

Theoretically, the velocity of traveling waves on the string should be: $v = \sqrt{\tau/\mu}$, where τ =tension and μ =mass of string/length. Standing waves can be created between points a distance L_n apart when there is an integer number of half-wavelengths between them i.e. $L_n = n\lambda/2$. For a fixed frequency, the tension in the string can be varied to produce standing waves of different wavelengths.

Combining the preceding equations we find that standing waves should be excited on the string when the tension satisfies $\tau_n = 4\mu f^2 L_n^{\ 2}/n^2$.

Experiment by applying or removing tension to or from the string (i.e slowly, carefully pull down or lift up on the mass hanger) and observe how different standing wavelengths can be excited as the tension varies. (Have a lab partner hold a contrasting object behind the string if it helps you see it better.) <u>Record your (qualitative) observations.</u> How does the wavelength vary with tension?

With tension adjusted so a number of half-wavelengths are excited (add or subtract masses to hold the tension) move your finger along the string and observe what happens to the standing waves. <u>Record your (qualitative) observations</u>. What happens if your finger is on a node? On an anti-node?

Now investigate the occurrence of standing waves quantitatively and systematically. Starting from the conditions you had previously when you created 8 anti-nodes, increase the tension by incrementally adding mass to the hanger until you've created 7 antinodes and record you data in the table as before. Continue in this way, creating 6, 5, 4, 3 and 2 clearly defined anti-nodes and record your data. (This will take patience and careful observation and adjustments!) For each such situation, record the number of half-wavelengths, measure L_n, and calculate the wavelength as described in the data table.

Show that: $\lambda = \frac{1}{f\sqrt{\mu}}\sqrt{\tau} = \frac{\sqrt{g}}{f\sqrt{\mu}}\sqrt{M} = C\sqrt{M}$, where C is a constant. <u>In your report explain why C should be a constant for this experiment.</u>

Use the computer (Excel or other program) to plot standing wavelength versus the square root of total hanging mass and determine whether the relationship is a good fit to a straight line. <u>Discuss your results</u>.

Find the slope of the line (using the computer) and determine the agreement with the theoretical prediction: *(You will have to make sure your units are the same when you make your comparison.)*

Conclusions:

Discuss the nature of standing waves and how they relate to unbounded propagating waves.

Waves on a string:

Mass m of piece of string:				
Total length L of a piece of string:			linear density: μ = m/L=	
Length between pulley contact point and reed attachment point (L): (should be 1.33m)				
Total number of observed loops n_T	Suspended Mass (total), M	Measured length for n loops (L_n)	Wavelength $\lambda_n = 2\,(L_n/n)$	$\sqrt{M}$

Include graph with axes labeled (including units) and analysis shown.

Slope of graph of λ vs. $\sqrt{M}$:	
Theoretical Prediction: $C = \dfrac{\sqrt{g}}{f\sqrt{\mu}}$	
Percent Discrepancy:	