Wave Optics: Interference and Diffraction (11/14/14) (approx. 2 h 15 min.)

Introduction

In previous optics labs we have described the properties of light simply in terms of rays, using the laws of reflection and refraction. This description works well for many phenomena, including image formation with lenses and mirrors, but for other circumstances it fails. In this lab we examine some phenomena which clearly require a different description. This alternative description is called *wave optics*, to distinguish it from the ray description which is called *geometric optics*.

Equipment

- optics bench
- metal screen & 5" by 8" card
- poster board (to expand screen)
- dim lamp (plus 1 for instructor)
- bench-edge clamp with vert. rod
- single & multiple slit sets
- red diode laser
- clip for poster board
- masking tape
- test tube holder on rod
- compact disk
- ruler
- optics caliper
- diffraction gratings
- pencils & erasers
- rod clamp

OPTIONAL: Pass small, sample holograms (on cart) around class for students to view.

Obey all instructions regarding laser safety: You do not want to shine a laser in anyone's eye!

Turn off the laser when you are not using it.

The direct beam is most hazardous: always use something (e.g. the screen) to block the laser beam before it leaves the lab table.

The reflected beam is less dangerous but should still be watched. If there are reflections make sure they do not shine in anyone's eye.

Theory

If the amplitudes of two light waves have the same sign at a point in space, then they interfere *constructively*: their amplitudes add, and the light intensity is brighter at that point. If they have opposite signs, however, they will interfere destructively: their amplitudes partially (or totally) cancel each other, and the light intensity is dimmer (or dark) at that point. It is these areas of strong and weak intensity that make up the interference patterns we will observe in this experiment.

Interference can be seen when monochromatic light (e.g. laser light) from a localized source arrives at a point on a viewing screen by more than one path. Because the number of wavelengths differs for paths of different lengths, the light waves can arrive at the viewing screen with a *phase difference* between their electromagnetic fields. If their amplitudes have the same sign they add *constructively* and result in light of increased intensity. If their amplitudes have opposite signs they add *destructively*, resulting in light of decreased intensity.

Diffraction can be observed, for example, when laser light travels through a vertical *slit* whose width, a, is small compared to its wavelength, λ. Light from different points across the width of the slit will take paths of different lengths to arrive at a point on the viewing screen. When light waves emanating from different points across the slit interfere destructively, intensity minima (dark regions) appear on the screen. Likewise, when these light waves interfere constructively, intensity maxima (bright regions) appear.

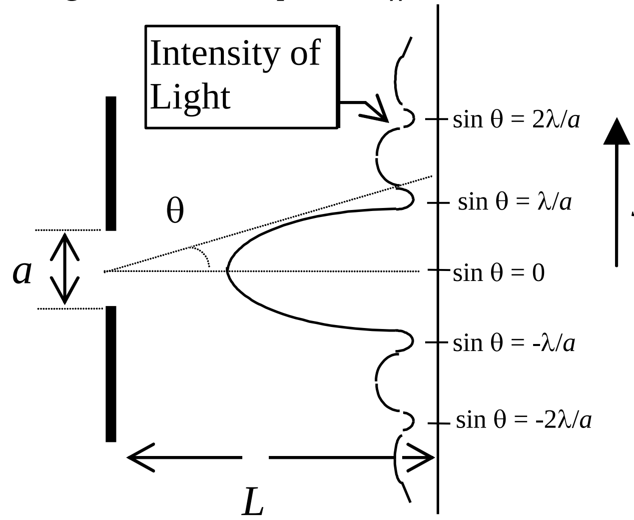

Figure 1: Fraunhofer Diffraction by a slit of width *a*. Graph shows intensity of light on a screen.

Single Slit Fraunhofer Diffraction:

For the case L >> a, we have far-field, or Fraunhofer diffraction. Figure 1 shows such a pattern (from above), as a graph of the intensity of light along a horizontal line on the screen. For a long slit of uniform width, a, it can be shown that the minima (dark lines) in the intensity pattern fit the formula: $a \sin\theta_m = m\lambda$, where m is a non-zero integer ($\pm1, \pm2, ..$), λ is the wavelength of the light and θ is the angle between the centerline of the slit and a position on the screen, as shown. The m^{th} dark spot on the screen is called the m^{th} minimum. Diffraction patterns for other shapes of openings (apertures) are more complex but also result from the same principles of interference.

Theoretically, in terms of the y coordinate on the screen, we see that the m^{th} minimum should be located at: $y_m = L \sin\theta_m \cong \frac{m\lambda L}{a}$.

Two-Slit Interference:

When laser light shines through two closely spaced parallel slits (Figure 2) each slit produces a diffraction pattern. When these patterns overlap, they also interfere with each other. We can predict whether this interference will be constructive (a bright region) or destructive (a dark region) by determining the path length difference, Δp, in traveling from each slit to a given spot on the screen.

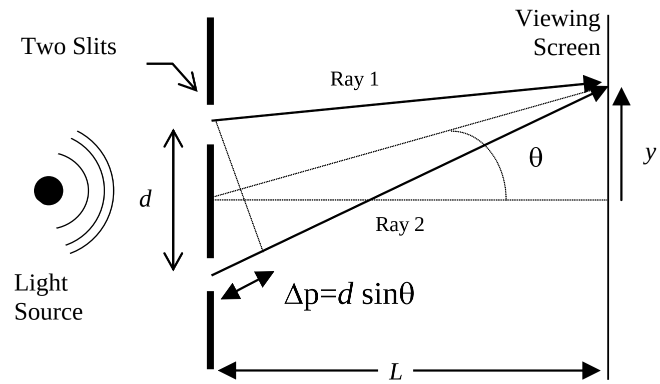

Figure 2: Interference of light from two slits. A maximum occurs when $\Delta p = m\lambda$ and a minimum when $\Delta p_m = (m + 1/2)\lambda$, where *m*=0,1,2,…

Intensity maxima occur when the light from the two paths arrives *in phase* (i.e. constructive interference) with an integer number of wavelengths difference between paths. That is, when the difference between path lengths, Δp, satisfies:

$$\Delta p_m = d \sin\theta_m = m\lambda, \text{ where } m = 0, \pm1, \pm2, \pm3, \ldots \ .$$

Intensity minima occur when the light from the two paths arrives *out of phase* (i.e. destructive interference) with a half-integer number of wavelengths difference between paths:

$$\Delta p_m = d \sin\theta_m = \left(m + \frac{1}{2}\right)\lambda, \text{ where } m = 0, \pm1, \pm2, \pm3, \ldots \ .$$

Since the distances y on the screen are much less than the distance, L, from the slits to the screen, we may take, to a good approximation, $\sin\theta \approx \tan\theta$. Therefore, in terms of the y coordinate on the screen in Fig. 2, we see that the m^{th} minimum should be located approximately at:

$$y_m = L \tan\theta_m \cong L \sin\theta_m \cong \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \qquad \text{where } m = 0, \pm1, \pm2, \pm3, \ldots .$$

From this formula it can be seen that the spacing between adjacent minima is: $\Delta y_m = y_{m+1} - y_m = \lambda L / d$.

Procedure

Part A: Diffraction From a Single Slit

The single slit set has slits on it of different widths. The widths are given on the accessory in mm.
Look through the slits, holding them very close to your eye. See if you can see the effects of diffraction.

Position the metal screen on one end of the optical bench and the diode laser on the other. Attach the long piece of poster board horizontally to the screen with the glossy side showing. Use a dim lamp for reading and writing.

THE ROOM SHOULD BE DARK FOR OBSERVATION OF DIFFRACTION PATTERNS.
DO NOT LOOK DIRECTLY INTO THE LASER OR AIM IT AT ANYONE!

Position the single slit accessory about 10 cm in front of the laser and switch the laser on. Rotate the slit accessory holder and the disc with the slits so that the 0.16 mm slit is vertical and in line with the laser beam. A horizontal diffraction pattern should appear on the screen. (You may need to adjust the direction of the laser beam with the adjustment screws until it is centered on the slit and a bright pattern appears on the screen.) Rotate the single slit accessory disc so the 0.08, 0.04, and 0.02 mm slits are in line with the laser beam and examine each of the patterns. Note how the width of the diffraction pattern varies with the width of the slit.

Measure (and record on p.5) the distance, L, from the slit to the screen. With the 0.04 mm slit in place, use the optics caliper and a ruler to measure and record the value of y corresponding to the center of the dark region on the viewing screen that is furthest from the center (i.e. from y=0 to y_m). Use the single slit equation for y_m to calculate an averaged value for a (see p.1). (The approximate wavelength of the diode laser is marked on it.) When you are confident you have recorded good measurements, erase any reference pencil marks you may have made. Time permitting, repeat the above procedure using the 0.16 mm slit.

The following two procedures are at the instructor's digression (i.e. time permitting)

Remove the slit set and pull a hair from your head, arm or eyebrow. Mount it vertically on the front of the laser using a piece of tape. Aim the laser at the screen and observe the diffraction pattern created by the hair. Make measurements and use the formula to estimate the thickness of the hair, a. (Although the hair is not a slit, light diffracts around its edges in a similar fashion.) Repeat with observations of your lab partners' hair. Answer the questions on page 5.

Observe the diffraction patterns for various types of aperture (squares, hexes, dots, holes and circular apertures). Do you perceive correspondences between symmetries of the apertures and their diffraction patterns?

Part B: Two-Slit Interference

Mount and align the double slit portion of your multiple slit set and observe the patterns projected on the viewing screen. Observe how the pattern changes with changing slit width, a, and slit separation distance, d.

For each pair of double slits, record the distance from the slits to the screen, L, and the total length, Δy, covered by several (n) minima. Calculate the average spacing between the minima and use that to calculate the slit separation, d. Answer the questions on page 6.

Part C: Transmission Diffraction Gratings (optional)

A transmission diffraction grating is an array of a large number of parallel slits, all with the same width and equal distances, d, between centers. The distance between slit centers is conventionally expressed in terms of the number of lines per unit length. So, for example, a grating having 100 lines per mm has a *grating spacing*, d, of 0.01 mm. Although the exact analysis gets very complicated for multiple slit diffraction, it turns out that the *principal* intensity maxima occur in the same directions as for the two-slit pattern, so the positions of the maxima are once again given by: $d\sin\theta = m\lambda$, where $m = 0, \pm1, \pm2, \pm3, \ldots$.

A pack of 4 diffraction gratings is available for further investigation. They can be mounted on lens holders using thumbscrews. The wide poster board screens may be necessary to see the bright spots. (It might also be necessary to move the grating closer to the screen.) Select one of the gratings to use to calculate the wavelength of the laser light using the above equation with $m = 1$. You can calculate d from the given number of lines per mm on the diffraction grating, and $\sin\theta$ from careful measurements of the distance from the grating to the screen, and the distance from the central principal maximum ($m = 0$) to the first principle maximum ($m = 1$). Show your calculations, and record your answer on page 7.

Part D: A Reflection Diffraction Grating (optional - more advanced)

In a *reflection* diffraction grating, the array of equally spaced slits of a transmission diffraction grating is replaced by an array of equally spaced ridges or grooves on a reflective substrate. The "grooves" on a compact disc (CD) have uniform radial spacing, and therefore act as a reflection grating. If a beam of monochromatic light is **incident normally to the radial line on the CD from which it is reflecting**, the reflected angles at which intensity maxima occur will be given still by: $d \sin\theta = m\lambda$, where d is the spacing between adjacent grooves. Also, to avoid distortion of distances between intensity maxima, the observation plane, i.e. the screen, should be parallel to the radial line on the CD from which reflection occurs. (As you can see, you'll have to use your spatial visualization abilities and give considerable thought to how you set up your laser, CD, and screen.) Test tube holders on rods, rod clamps, and a rod which can be inserted vertically into a desktop rod holder, have been provide to aid in your setup. The rest is up to your ingenuity!

Since λ is known, and you can calculate $\sin\theta$ from measurements, you can calculate the distance, d, between grooves on a CD. Make a sketch of your setup, show your calculations, and record your estimate of d on page 7.

Part E: Viewing a hologram (optional – instructor may choose to pass around for viewing)

A hologram records a virtual image that contains both amplitude and phase information. When looking at a three dimensional object, both the amplitude and phase of light vary as you look at the object from different angles. Since a hologram, unlike an ordinary photograph, records phase information, the image of the hologram changes with viewing angle, and it appears to be three dimensional.

Data, Calculations and Analysis

Part A: Single Slit Diffraction Data and Analysis

Specified width of first slit: a = ________ Intensity Pattern Sketch:

L = ___________________

λ = ___________________

Diffraction Order, *m*	Distance to minimum, y_m	y_m / L	Angle, θ_m in *radians*	sin θ_m	$a = \left(\dfrac{m\lambda}{\sin\theta_m}\right)$

Calculation of slit width (show your work):

Calculated slit width = ________________________________

Specified width of second slit: _______ Intensity Pattern Sketch:

L= ___________________

λ = ___________________

Diffraction Order, m	Distance to minimum, y_m	y_m / L	Angle, θm in radians	sin θm	$a = \left(\dfrac{m\lambda}{\sin\theta_m}\right)$

Calculation of Slit Width (show your work):

Calculated Slit Width = ______________________________

Estimated width of a hair (Describe your measurements, show your work):

Questions:

How does the distance between diffraction minima change (increase or decrease) as the slit width is decrease?

Why don't you see diffraction patterns when light is sent through a large hole?

Part B: Two Slit Interference Data and Analysis

First specified slit spacing used: d =_______ Intensity Pattern Sketch:
L= ________________ a =________
λ = _______________

m (m=0 at center)	Distance to minimum, y	$y/L \approx \sin\theta$	$\left(m+\frac{1}{2}\right)\lambda = \Delta p$	d = $\frac{\Delta p}{\sin\theta}$

Are the distances between minima about equally spaced? ________ Should they be? (Explain)

Calculation of average slit spacing of n minima spread over a distance Δy:

Calculated average slit spacing: (Δy /n) _______________________________

Second specified slit spacing used: d= _______ Intensity Pattern Sketch:
L= ________________ a= _________
λ = ________________

m (m=0 at center)	Distance to minimum, y	$\frac{y}{L} = \sin\theta$	$\left(m+\frac{1}{2}\right)\lambda = \Delta p$	d = $\frac{\Delta p}{\sin\theta}$

Calculation of average slit spacing over n minima:

Calculated average slit spacing: (Δy /n) _______________________________

Questions:

In addition to the pattern of dark spots is there another visible pattern in the brightness of the bright spots?

If so, to what (specifically) do you attribute this pattern?

Part C: Transmission Diffraction Grating Calculation (optional)

Calculated value of d =________

Calculated value of sin θ = ______________

Calculated value of $\lambda = d\sin\theta$ = ______________

Compare your calculated value of λ to the nominal value given on the laser and calculate the

% difference: ___________________(use the correct sign!)

Part D: Reflection Diffraction Grating (CD) Calculation (optional)

Make a sketch of your setup:

Explain the factors you had to consider to set up the laser and the CD to get a reflected diffraction pattern such that the equation $\lambda = d\sin\theta$ could be used to calculate d, the radial spacing between grooves.

Calculated value of groove spacing: ______________

How does this compare to 1600 nm, the standard radial spacing between grooves on a CD?