Charging and Discharging a Capacitor (approx. 2 h 20 min.) (5/16/12)

Introduction

A capacitor is made up of two conductors (separated by an insulator) that store positive and negative charge. When the capacitor is connected to a battery current will flow and the charge on the capacitor will increase until the voltage across the capacitor, determined by the relationship C=Q/V, is sufficient to stop current from flowing in the circuit. Figure 1 shows a circuit that can be used to charge and discharge a capacitor.

Equipment

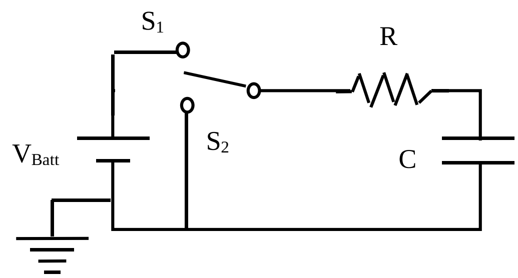
- PB-60 proto-board & power adapter
- 2 capacitors (100 μF)
- single-pole, double- throw switch
- 750 interface & Data Studio
- proto-board wires
- voltage sensor
- resistor (10 kΩ)
- alligator clips
- multi-meter

For whole class: capacitance meter: spare fuses; Phillips screwdriver

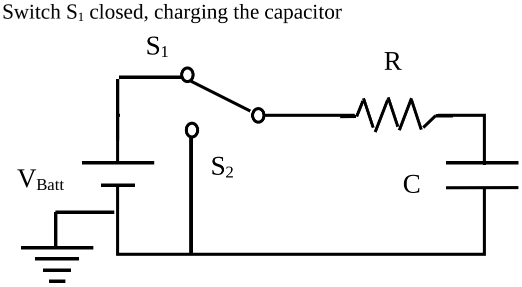
Theory

Charging the capacitor

Circuit with two position switch, S_1 and S_2



Switch S_1 closed, charging the capacitor



Switch S_2 closed, discharging the capacitor

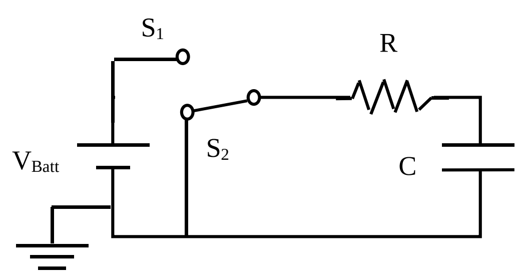


Figure 1: Circuit to charge and discharge a capacitor

Before the switches are closed, there is no charge on the capacitor. When switch S_1 is closed, current will flow in the circuit as the capacitor is charged. According to Ohm's Law, the voltage across the resistor will be

$V_R = IR$

while the voltage across the capacitor will be given by

$V_C = Q/C$.

By Kirchhoff's Rule the voltage changes around the circuit must add to zero so

$V_{batt} - V_R - V_C = V_{batt} - I\,R - Q/C = 0$

When the capacitor charges the charge, Q, starts at zero and there is no voltage on the capacitor. This means the current initially flows at its maximum rate ($I_{Max} = V_{batt}/R$ when Q=0). However, as the flowing current charges the capacitor, the voltage on the capacitor increases. This voltage opposes the flow of more charge and the current begins to decrease. The *rate* at which the capacitor charges slows as the current decreases -- as more and more charge builds up the current becomes smaller and smaller. The current *decreases exponentially* -- it asymptotically approaches zero for longer and longer times. Similarly the charge *increases* exponentially -- it keeps growing but at a slower and slower rate and asymptotically approaches (but never actually reaches) its maximum value. It would take an infinite amount of time to actually reach the maximum value since the rate of increase (current) becomes smaller and smaller as time goes by. If we take into account the fact that current and charge both vary with time, the equation obtained by applying Kirchhoff's Voltage Rule around the charging circuit becomes:

Equation 1: Kirchhoff's Rule for a charging capacitor: $V_{Batt} - V_R - V_C = V_{Batt} - I(t) \cdot R - \dfrac{Q(t)}{C} = 0.$

The exponentially decreasing current and increasing charge are described by:

Equation 2(a): I(t) for the charging capacitor

$$\mathrm{I(t)} = \frac{\mathrm{V_{Batt}}}{\mathrm{R}}\left(e^{-t/RC}\right)$$

Equation 2(b): Q(t) for the charging capacitor

$$\mathrm{Q(t)} = \mathrm{CV_{Batt}}\left(1 - e^{-t/RC}\right)$$

The number *e* (Euler's Number also known as the base of natural logarithms) appears in any equation in which the rate of increase or decrease of a quantity depends linearly on the amount of that quantity. In this case the rate at which the capacitor charges depends on the current, which decreases as the charge (and hence the voltage) on the capacitor increases. Theoretically, it takes an infinite amount of time for the current to actually decrease to zero or for the capacitor to become fully charged. This is a property of exponential functions. The exponential behavior is characterized by the *time constant,* τ :

Definition of time constant:

$$\tau = \mathrm{RC}.$$

The time constant has units of seconds. The larger the product RC the longer it will take the capacitor to charge to any fraction of its maximum value. The time required to charge to $\left(1 - e^{-1}\right)$ of the maximum value is exactly one time constant, RC. Euler's number, *e* is an irrational number equal to $2.718281828459045235360287 47 \cdots etc.$

Discharging the capacitor

If we wait for until several time constants have passed, the capacitor will become nearly fully charged. At that time the current is nearly zero, the voltage on the capacitor is approximately equal to the voltage on the battery, V_{Batt}, and its charge Q_0 is given by $Q_0 = CV_{Batt}$. Now we can change the switch from position S_1 to S_2. The current will flow through the resistor to ground, discharging the capacitor. Around this loop the sum of voltages is now given by

Equation 3: Sum of voltages around the discharging circuit:

$$\mathrm{V_R} + \mathrm{V_C} = \mathrm{I(t)} \cdot \mathrm{R} + \frac{\mathrm{Q(t)}}{\mathrm{C}} = 0$$

The voltage on the capacitor acts to "push" the current but as the current flows the capacitor discharges and the current slows down. Thus the rate of discharge slows as time goes by. Both the current and charge *decrease exponentially* with time:

Equation 4(a): I(t) for discharging capacitor

$$\mathrm{I(t)} = \frac{V_C}{R} e^{-\mathrm{t/RC}} = \frac{\mathrm{Q_0}}{RC} e^{-\mathrm{t/RC}}$$

Equation 4(b): Q(t) for discharging capacitor

$$\mathrm{Q(t)} = \mathrm{Q_0} e^{-\mathrm{t/RC}} = \mathrm{CV_{Batt}} e^{-\mathrm{t/RC}}$$

The same time constant, $\tau = RC$ is used to characterize the discharging cycle but now the time RC describes the time it will take the charge to decrease to $1/e = e^{-1} = 0.367879$ times of its initial value.

Procedure

Before building your circuit, flip the PB-60 proto-board over and carefully examine the pattern of connections, noting where gaps occur. Attach the power transformer to the adapter on the PB-60 proto-board and **construct the circuit** in Figure 1 using the +5 V output, a 100μF capacitor, a 10kΩ resistor and a double-pole, single throw knife switch. With these values, the product of R times C is about 1 second. (Note: If there is an arrow on the capacitor, it should point from higher voltage to the lower voltage connection.) Use the banana-to-alligator cords to connect the proto-board to the switch terminals. As you set-up the circuit, use a multimeter, set as an ohmmeter, to check continuity in your circuit. Then use it as a voltmeter and make sure you understand how the circuit works. **ASK YOUR INSTRUCTOR TO CHECK YOUR CIRCUIT BEFORE PROCEEDING.**

Preparing the data acquisition hardware: On your computer desktop, click on Data Studio, select Create Experiment, and choose Voltage sensor. Connect the sensor to the PASCO 750 interface as shown. Select Graph from the display menu. Connect the voltage leads across the capacitor. By measuring the voltage across the capacitor you can measure the charge since Q=CV.

To collect data you should **start with the switch in the grounded position** (S_2 in Figure 1). You can use an extra wire to **ground both sides of the capacitor** to make sure it is fully discharged before you start.

Click **Start** on the program. You should see a horizontal line at about zero volts. After a couple of seconds flip the switch to the charging position. You should see the voltage continually increase. If the voltage increase is too fast or too slow stop and re-adjust the time scale by clicking and dragging on the time axis. Repeat until you are sure you can make a good, readable data plot.

Optimize the graph: right click on the graph and you see several functions: “scale” will auto-scale the graph, “in/out” zooms, “measure” will create a cross-hair cursor. You can select or delete certain graph. In addition you can click and drag on the number on the axis and can zoom in/out in this way.

Once you have adjusted the time scale and data collection rate you can begin taking data.

Start with the switch in the S_2 position and ground the capacitor to discharge it fully.

Click start on the program and let it run for a while to establish a flat zero volt baseline.

Flip the switch to S_1. Let the program run as the voltage rises.

When the voltage reaches the maximum **let the program run** so that there is a flat line at the maximum voltage --- the charge on the capacitor has now reached its maximum and the current has stopped flowing.

Keep the program running and flip the switch back to S_2. Now the capacitor will discharge. Let the program run long enough to again establish a flat zero volt baseline.

If you have recorded data for several cycles, choose the best looking charge/discharge graph (you can delete the other graphs, if you like).

From your graph, you will **determine the time constant** of the circuit for both the charging and the discharging portions of the curve. From the time constant and the measured value for your resistance, you will derive the **capacitance**. (See ***Data Analysis*** section NOW for detailed instructions)

IMPORTANT: STOP and analyze your data before proceeding to the next section! You will need to have the data visible on the computer in order to analyze it.

Capacitors in parallel and in series

Repeat the data collection for two capacitors in series *and* for two capacitors in parallel. Demonstrate that the sum of the capacitance conforms to the formulae derived in class:

Capacitors in Parallel: $C_{equivalent} = \sum C_i = C_1 + C_2 + C_3 + \ldots$

Capacitors in Series: $\dfrac{1}{C_{equivalent}} = \sum \dfrac{1}{C_i} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \ldots$

Data Analysis

Be sure and complete your analysis as you complete each part of the lab

Understanding the time constant.

The "time to charge" or the "time to discharge" is theoretically infinite. The charging and discharging times are therefore *characterized* by a time τ=RC, called the time constant. From the equations for voltage as a function of time we can figure out what the voltage actually is at these times.

When the capacitor is charged its charge increases according to Equation 2 and therefore its voltage as a function of time is given by the equation:

$$V_C(t) = \frac{Q(t)}{C} = V_{Batt}\left(1 - e^{-t/RC}\right) \qquad \text{(Charging)}$$

The time constant, τ=RC, is defined as the time when the charge reaches the value given by setting the time t equal to the value RC. At this time the voltage reaches a certain fraction of the battery voltage:

$$V(\text{at } t = RC) = V_{Batt}\left(1 - e^{-1}\right) = 0.632\ V_{Batt} \qquad \text{(Charging)}$$

To find the time constant from the graph simply find the time it takes the voltage (and hence charge) to increase from zero to (1 - e^{-1}) times its final value. You will find this time by analyzing the graph in the following way:

- Right click on graph and select measure. You should see a cross-hair cursor, which gives you the time and voltage (t,V);
- Move cursor to the exact point where the voltage starts to rise. Note the time t_1.
- Move the cursor to the point where the voltage is leveled off and maximal. Note V_{max}.
- Move the curser along the graph to the level where the voltage reads $0.632V_{max}$. Note the time t_2.
- Calculate the time interval $\Delta t = t_2 - t_1$ between V=0 and 0.632 V_{max}, which is the **time constant**.

When the capacitor is discharging, the charge decreases according to Equation 4 and therefore its voltage obeys the equation:

$$V(t) = V_{Batt}\, e^{-t/RC} \qquad \text{(Discharging)}$$

The time constant is: τ=RC. At this time the voltage reaches a certain fraction of the battery voltage given by:

$$V\ (\text{at } t = RC) = V_{Batt}\left(e^{-1}\right) = 0.368\ V_{Batt.} \qquad \text{(Discharging)}$$

To find the time constant from the graph simply find the time it takes the voltage to decrease from its maximum value to (e^{-1}) times that value, by using a similar procedure as described above.

Once you have found the time constant, complete the calculations in the data table to compare the measured time constant to the theoretical value. You can use the known value of the resistor to calculate an experimental value for the equivalent capacitance for the capacitors in parallel and series. To calculate the theoretical C_{equiv} use the theoretical equations for the series and parallel circuits on page 3.

For your report

For your report, write a cover page with brief introduction and include data table, graphs and analysis. All graphs should have axes labeled with units. The analysis, which gives you your time constant, should be presented clearly.

Calculations given in the data table should be shown below the graphs or on a separate page. Summarize your experimental results and discuss errors or discrepancies and their possible sources.

Discussion and Questions

Discuss why the time required to charge and discharge changes with changes in capacitance and resistance (How much charge is required to reach the final voltage? At what rate does the charge flow? What controls the current?)

How does the time constant change when the capacitors are in parallel? In series? How does the capacitance change?

Analyze the time dependence of the solutions in Equations 2 and 4.

- Look at Equation 2(b) and Equation 4(b) for the charge (or voltage since VC=Q/C) at time t=0. Substitute t=0 in the equations. Does each equation give the appropriate initial value for a charging or discharging capacitor?
- Look at Equations 2(a) and 4(a) and determine what is happening to the current at time t=0.
- Look at the same equations at very long time (substitute a very large value or imaging what the value reaches in the limit t=infinity). Do the equations reach the appropriate limits at very long times?

Write two paragraphs describing in your own words what is happening to the charge on the capacitor, the voltage on the capacitor, and the current in the circuit as the capacitor is 1) charging and 2) discharging. Comment on whether the time dependence shown by the equations agrees with the expected time dependence of the charge and current in the circuit as it charges or discharges.

Calculus Based Questions: Understanding the equations in more detail.

Equations 1 and 3 describe the charging and discharging of a capacitor. The solutions to these equations are Equations 2 and 4, respectively.

Equation 2(b) describes the charge as a function of time as the capacitor is charged.

- Find the currents for the charging capacitor by calculating the function I(t)=dQ/dt for this case. (Take the derivative of Equation 2 to find I(t).) Compare to Equation 2(a).
- Substitute the function Q(t) of Equation 2(b) and the function for I(t) which you have just found into Equation 1, show that Equation 1 does give zero. This proves that Equations 2(b) is a solution for this circuit.

Equation 4 describes the charge as a function of time as the capacitor is discharged.

- Find the currents for the discharging capacitor by calculating the function I(t) = dQ/dt for this case. (Take the derivative of Equation 4(b) to find I(t).)
- By substituting the function Q(t) of Equation 4(b) and the function for I(t) which you have just found into Equation 3 show that Equation 4 is indeed a solution to the voltage equations for a discharging capacitor. (Show Equation 3 does give zero).

Single Capacitor

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitance C used

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	

Parallel Capacitors

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitances used	C_1	C_2

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculation for C_{equiv}			
Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	
Experimental C_{equiv} (From τ =RC)		Percent Diff. (between experimental and calc. C_{equiv})	

Series Capacitors

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitances used	C_1	C_2

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculation for C_{equiv}			
Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	
Experimental C_{equiv} (From τ =RC)		Percent Diff. (between experimental and calc. C_{equiv})	