Resistances in Parallel and in Series (12/20/11)

Introduction

In this lab you will investigate simple DC (direct or constant current) circuits made with a DC power supply and resistors. You will be able to measure voltage, current, and resistance for different components in your circuits.

Equipment

- PB-60 protoboard with connector and DC power supply
- resistors (100Ω to 1000Ω)
- protoboard wires (20 or 22 gauge)
- multimeter with alligator clip or point probes

For class as a whole: Phillips head screwdriver and spare fuses (0.5amps, 250V, fast acting) for multimeters

Theory

To understand how current (I), voltage (V), and resistance (R) are related in a circuit, we can use a few basic relationships:

Ohm's Law $V = I R$

Resistance is a measure of how difficult it is for current to flow through a circuit element (a wire, a resistor, etc.). "Ohmic" devices have a linear relationship between voltage and current.

Kirchhoff's Rules

JUNCTION RULE: The sum of currents going into any node (i.e. junction) in a circuit must equal the sum of currents leaving the node.

LOOP RULE: The sum of voltage changes around any loop in a circuit is zero.

"Kirchhoff's Rules" arise from the principles of conservation of charge and conservation of energy respectively. The first rule simply says that since current is moving charge, the number of Coulombs per second (Amperes) going into any junction of a circuit must be the same as the total current leaving that junction. For the node shown in the top frame of Figure 1 this can be written as

$$I_A = I_B + I_C$$

The second rule says that the net change in potential energy per unit charge (that is the change in voltage) must be zero when you make a complete loop around any path in a circuit. The bottom frame of Figure 1 shows a circuit with two loops. Using the currents defined in the top frame, the changes in voltage around the loops can be written

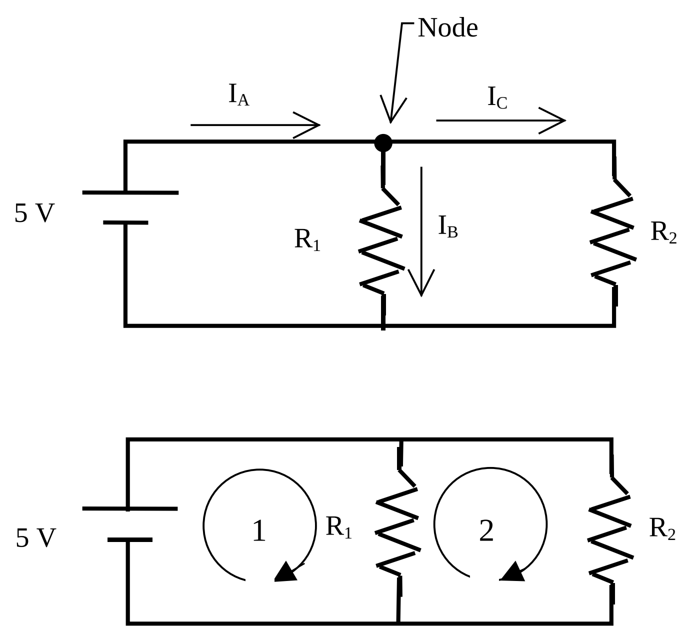

Figure 1: The current going into a node must equal the current leaving a node (Top). The voltage change going around any closed loop must be zero (bottom).

$$5\text{ V} + V_{R1} = 5\text{ V} - I_B R_1 = 0 \qquad \text{(loop 1)}$$

$$V_{R1} + V_{R2} = +I_B R_1 - I_C R_2 = 0 \qquad \text{(loop 2)}$$

Here we also use Ohm's Law which says that the voltage across a resistor should <u>decrease</u> by V=IR <u>in the direction of the current</u>. Notice that we follow each loop in a single direction. For loop 2, we go around the loop in the direction opposite to I_B but in the same direction as I_C, and so the voltage increases by $I_B R_1$ but decreases by $I_c R_2$. The three equations determined from Kirchhoff's Rules provide enough information to allow us to solve for the three currents I_A, I_B and I_C.

Effective Resistance for Resistors in Parallel and in Series:

The relationship between V and I for a circuit with several resistors can be written in the same form as Ohm's Law but with an *effective resistance* representing the resistors: $V = I\,R_{effective}$.

Resistors in series have *the same current going through each of them*, so the total resistance increases with the addition of each resistor. The effective resistance of several resistors in series is given by

$$R_{effective} = R_1 + R_2 + R_3 + \ldots \qquad \text{(in series)}$$

Resistors in parallel have *the same voltage across each of them*. The circuit in Figure 1 shows two resistors in parallel. Because the current can split up among the different paths, the effective resistance decreases. It can be shown that the effective resistance of several resistors in parallel is given by:

$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{(in parallel)}$$

Measuring Voltage, Current and Resistance with a Multimeter:

***Voltage**:* When using a multimeter as a voltmeter to measure voltage, the probes are attached across two points in the circuit and the meter measures the voltage *difference* ΔV between those two points. The ideal voltmeter has a very high resistance so that it draws negligible current from the circuit.

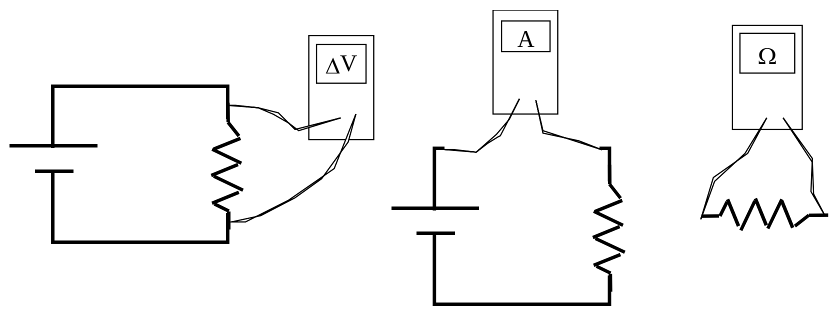

Figure 2: A voltmeter measures the voltage difference across two points in the circuit. An ammeter must be inserted into the circuit in order to measure current. When an ohmmeter is used to measure resistance, the circuit element must be disconnected from the rest of the circuit.

***Current**:* When a multimeter is used as an ammeter to measure current (in amperes or "amps"), it must be inserted *into* the circuit at the point where current to be measured passes through it. In other words, the circuit must be broken (i.e. opened) at that point. The ideal ammeter has very low resistance so that it doesn't significantly affect the voltage or current in the circuit (i.e. *There should be essentially no voltage difference across it.*).

WARNING: It is possible to damage an ammeter by passing too much current through it. Make sure it is properly connected (in series with a resistor as shown in Figure 2***). If too much current goes through the ammeter it will blow a fuse! As a precaution, use the 10A setting first to check that the current is less than 0.4A before using the 400 mA or 40 mA setting to make a more precise measurement!***

***Resistance**:* When using a multimeter as an ohmmeter to measure resistance of a circuit element (in units of ohms (Ω)) the circuit element should be removed from the circuit to measure its resistance, otherwise you may not be measuring the resistance of that element alone, but rather the effective resistance of it and other components it is connected to.

Understanding Resistor Codes:

Resistors are often color coded with their values. There are usually four bands of color. The first two bands represent two digits and the third band represents additional powers of ten. The fourth band is either silver or gold, indicating the value is good to 10% or to 5%. The numbers represented are:

Color	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
Value	0	1	2	3	4	5	6	7	8	9

A resistor with bands of Brown, Blue, Red and Gold would signify the three digits 162 and have a value of $16 \times 10^2 = 1600$ Ohms. Practice using your multimeter to measure resistance until you understand the color codes on your resistors.

Procedure

Series Circuit

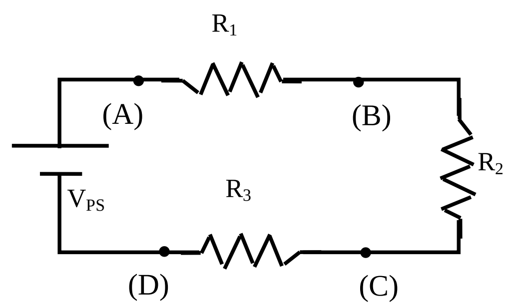

Figure 3: Series Circuit

Choose three resistors of different values between 100 and 1000 ohms. Use your multimeter as an ohmmeter to measure the three resistances and record them in a data table.

Examine the bottom of your protoboard to understand how the rows and columns are connected.

Connect the power supply to the protoboard connector.

Using the 5 V terminals of the connector, construct the series circuit shown in Fig. 3.

Measure the voltages across each resistor. The current should flow from the positive terminal of the power supply to the negative and the voltages should drop across each resistor in the direction of the current. If you are going around the loop in a clockwise direction, make sure your multimeter's negative probe contacts the "same" side of each resistor. (For example across R_1 point (A) should be the negative terminal, across R_2 it should be point (B), across R_3 point (C) should be the negative terminal, and across the power supply point (D) should be negative. Record these values as $V_{AB,}$ V_{BC}, etc.

Now you will use your multimeter as an ammeter to measure current. **BE SURE TO START ON THE 10A SETTING SO YOU DON'T BLOW THE FUSE ON YOUR METER.** <u>**Do not use it the same way as you would a voltmeter!**</u> ***The meter must be inserted into the circuit to measure the current***. To measure the current at point (A), for example, you would unplug the wire from R_1 and connect the red lead of the ammeter to the +5V terminal of the power supply. The other lead of the ammeter is then connected to the wire from R_1 that had been connected directly to the power supply. The current you wish to measure goes through the ammeter but is limited by also having to go through the resistors. Refer again to Figure 2.

NOTE: Current flowing <u>into</u> the red terminal of the multimeter reads positive.

Measure the currents at points A, B, C and D. BE SURE TO CHANGE THE SCALE ON YOUR MULTIMETER TO MATCH YOUR MEASUREMENTS. Record as I_A, $I_{B,}$ etc.

Use Ohm's Law to predict the voltage drop through each resistor and compare to the measured values. The circuit should obey Ohm's Law to within about 10 percent. ($V_{PS} = I\ R_{effective}$). If this is not the case the circuit may not be constructed properly, your meter may be inaccurate or you may not be using it properly. Stop and seek help if this is the case.

Calculate the effective resistance for all three resistors (see above) and compare to the value determined from Ohm's Law: $V_{PS}=I_{PS}R_{effective}$.

Apply Kirchhoff's Voltage (i.e. Loop) Rule by writing an equation for the voltages around the single loop of this circuit. Write it first with symbols, then use your numbers to check this rule.

Parallel Circuit

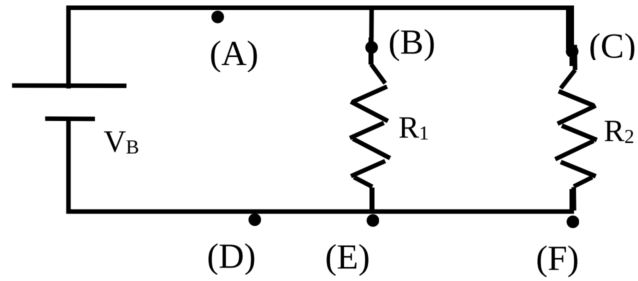

Figure 4: Parallel Circuit

Choose the two resistors with the highest resistance. Measure the values with your multimeter and record them.

Construct the parallel circuit shown.

Measure and record the voltages across the power supply and the resistors.

Insert your multimeter within the circuit at points A, B, C and D (using the 10 A setting first) and record the currents.

Do the resistors each obey Ohm's Law?

Solve for $R_{effective}$ for two resistors using the given expression for resistors in parallel, then calculate the effective resistance of the two resistors in *parallel* in your circuit and determine whether the voltage across the battery is equal to the current from the battery times the effective resistance.

There are two loops in this circuit. Going around each loop clockwise, write Kirchhoff's voltage loop equations, first with symbols, then with your measured values.

There are two nodes in this circuit. Draw pictures of the nodes and show the currents coming in and out. (Label them I_A, I_B, I_C or I_D) Write the equations for Kirchhoff's Rules as it applies to these nodes, first with symbols, then with your numbers to see that your measurements are consistent with the junction (node) rule within experimental accuracy.

For your report:

Your report should include a title page with summary, sketches and data tables, equations. The final summary should address how well Ohm's Law or Kirchhoff's rules hold and compare experimental values to theoretical predictions. (Comparisons should include typical percentage error). Discuss possible sources of discrepancy.

Deriving the effective resistances: (optional)

For the series circuit, use Kirchhoff's equation for the voltages then for each voltage substitute the equation $V_{PS} = I\,R_{effective}$ to re-derive the expression for the effective resistance of resistors in series.

For the parallel circuit, use Kirchhoff's current equation for the top node on this circuit and then for each current substitute the relationships $I = V / R$ to re-derive the expression for the effective resistance of parallel resistors.

Series Circuit: Sketch the circuit and label resistors and points along the circuit where measurements where made. Record data below and label each number to correspond to your sketch (For example V_{AB} = 1.56V or I_B = .046 Amps.)

Record Data Here, then transcribe below for your calculations:

Power Supply:	V_{PS} =	R_1=	R_2=	R_3=
Voltages Measured:	V_{AB}=	V_{BC}=	V_{CD}=	
Currents Measured:	I_A=	I_B=	I_C=	I_D=

Data (insert symbol and value from above) and calculations:

	Resistor	Current for this Resistor	Voltage from Ohm's Law	Measured Voltage	% Difference
Resistors:	R_1=	I_A=			
	R_2=				
	R_3=				

Effective Resistance	R_{eff}= V_{PS}/I_{PS}	
Theoretical Value	R_{eff}	
% Difference		

Kirchhoff's Rules for voltages around the series circuit (write out using symbols, then show the calculations using your numbers):

Parallel Circuit: Sketch the circuit and label resistors and points along the circuit where measurements were made. Record data below and label each number to correspond to your sketch (For example V_{AB} = 1.56V or I_B = .046 Amps.)

Record Data Here, then transcribe below for calculations:

Power Supply:	V_{PS}	R_1=	R_2=	
Voltages Measured:	V_{BE}=	V_{CF}=		
Currents Measured:	I_A=	I_B=	I_C=	I_D=

Data (insert symbol and value from above) and calculations:

	Resistor	Current for this Resistor	Voltage from Ohm's Law	Measured Voltage	% Difference
Resistors:	R_1=	I_B=			
	R_2=				

Effective Resistance	$R_{eff}= V_{PS}/I_{PS}$	
Theoretical Value	R_{eff}	
% Difference		

Write the following, using symbols, then show the calculations using your numbers.

Kirchhoff's Rules for Voltages around the two loops:

Loop 1: Loop 2:

Kirchhoff's Rules for Currents going into and out of a node: