The Current Balance:
Measuring the Force between Two Current-Carrying Conductors (about 2.5h) (8/5/13)

Introduction

Parallel, current-carrying wires interact with each other. Although the interaction is relatively weak, it is strong enough to be measured in a delicate introductory lab. The design of the apparatus used is similar to the experimental setup that defines the SI unit of electrical current, the ampere (A), as "that unvarying current which, if present in each of two parallel conductors of infinite length and one meter apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} Newtons per meter of length".

Equipment

current balance	laser & DC adapter	ruler	screen (poster strip)
fractional weight set	10 A power supply	2-meter stick	3 banana-to-banana leads (2 red, 1 black)
coin	magnetic compass	pencil	
vertical rod	masking tape	caliper	

For class as a whole: small adjustable wrench; pliers; Phillips screwdriver; flathead screwdriver

Safety Issues:

- Lasers are potentially dangerous. Even a reasonably low power laser can cause damage to your eyesight. Under no circumstances should you direct the laser beam toward a classmate or yourself. Always direct the light away from the rest of the class. The beam should be blocked with the manual beam blocker on the end of the laser or turned off when not being used for measurement.
- The work performed in this lab uses power supplies outputting voltages < 12 VDC, so serious electrical shocks are unlikely. However, it is always good practice to turn off the current through the circuit when making any modifications to the experiment and when not using it. This will minimize the possibilities of electrical shock and burns due to hot wires.
- 2-meter sticks are potentially eye hazards when carried around. Be careful!

Before the Lab

Read and understand the theory section of this lab. Answer the questions at the end of the theory section. Read through the remaining sections of the lab before setting up the equipment.

Theoretical Model

When a charged particle moves in a magnetic field, it experiences a force due to its motion (known as the Lorentz Force) given by the expression:

$$\vec{F} = q(\vec{v} \times \vec{B}), \tag{1}$$

where q is the charge on the particle, v is the velocity of the particle and B is the magnetic field. If the charged particle is an electron and the direction of motion of the charge is perpendicular to the magnetic field (on average), then the cross product reduces to:

$$F = evB. \tag{2}$$

Generalizing this expression for a current-carrying wire in a uniform, perpendicular magnetic field is accomplished by adding the force on each of the moving charge carriers (electrons). In Fig. 1, we count all of the charged particles in the volume of wire $V=AL$, where L is the length of the wire and A is the cross-sectional area of the wire. If the density of charge carriers in this volume is n (#/m^3), then the total number of charged particles in this volume is $N=nAL$. Equation 2 can then be generalized to give the average Lorentz force acting on a length L of wire carrying a constant current,

$$F=nALev_dB, \tag{3}$$

where B is the magnetic field throughout the region of the wire, and v_d is the average or "drift" velocity of the conducting electrons.

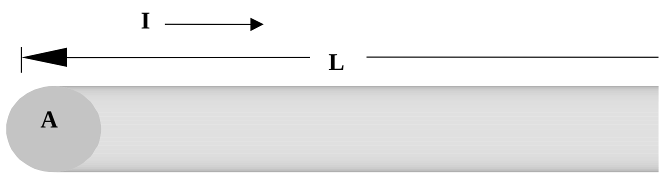


Fig. 1. Schematic diagram of a current carrying wire.

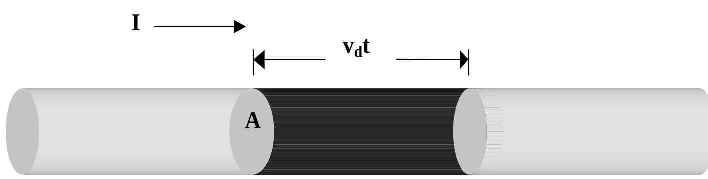


Fig. 2. Schematic diagram of the volume used to calculate the current.

The current is the amount of charge that passes any cross-section of the wire per unit time. In this case, we determine the number of charged particles (number of electrons) contained in the shaded region. The charge in this region passing any cross-section in time t is:

$$Q=nev_dtA,$$

where the parameters are as defined above. The current in the steady state is:

$$I=\frac{Q}{t}=\frac{nev_d tA}{\mathrm{t}}=nev_dA. \tag{4}$$

The force acting on the current-carrying wire can be rewritten by substituting eq. (4) into eq. (3) to get:

$$F=(nAev_d)LB=ILB, \tag{5}$$

which is the force acting on a wire of length L carrying uniform current, I, in a uniform, perpendicular magnetic field, B.

Suppose the source of the magnetic field, B, is a second wire, parallel to, and carrying the same current, I, as the first wire. If the current-carrying wires are approximated as lines (i.e. negligible diameter compared to their separation), then the magnetic field created by the second wire, at a distance *r*, can be derived by applying Ampere's Law: $\oint \vec{B} \bullet d\vec{l} = \mu_o I$.

Constructing a Gaussian surface as shown in Fig. 3, the path integral reduces to $2\pi rB$. Therefore, the magnetic field at a distance r from the wire is given by:

$$B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}. \tag{6}$$

Since the first wire is treated as a line a distance r away, we place this result in eq. (5) to yield:

$$F = ILB = IL\frac{\mu_o I}{2\pi r} = \frac{\mu_o L}{2\pi r} I^2, \tag{7}$$

where L is the length of the first wire and r is the centerline-to-centerline distance between the wires. This is the theoretical expression we will use to predict the slope of F plotted vs. I^2 in today's laboratory.

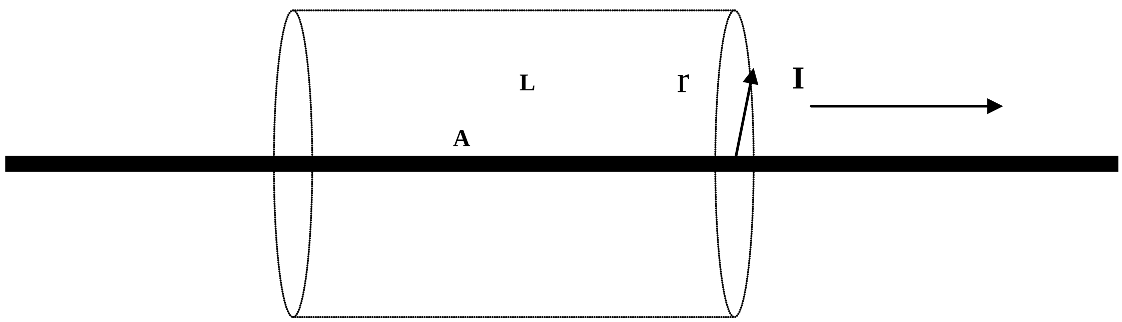


Fig. 3 Image showing the "Gaussian" surface used to determine the magnetic field from Ampere's Law.

Pre-lab Questions (Good for Discussion!):

Based on eq. (7), if the force between two current-carrying wires increases by a factor of four and the separation distance between the wires remains unchanged, by how much must the current in the wires have increased?

If the distance between two current-carrying wires decreases by a factor of one-third and the current does not change, by what factor should the force between the wires change according to eq. (7)?

The diagram shown in Fig. 4 (below) is, of necessity, two-dimensional. The direction of the current in the lower current balance rod is from right to left in Fig. 4. As shown in Fig.4, with the "upper" wire running parallel to the stationary wire, what is the direction of this magnetic field at the position of the upper wire? (i.e. "into" or "out of" the page)

Method of Operation

The current balance will be set up to pass the currents in opposite directions so the rods will repel. Once the equilibrium distance between them is determined with no current flowing and no weight added, weight will be added to the moveable upper rod, thereby lowering it. Enough current is then passed through the rods to lift the upper rod back up to the initial equilibrium position. The repulsive force necessary to do this must equal the weight that was added to the upper rod. The experimental relationship between F and I^2 thus determined will then be compared to the relationship predicted by equation (7) using values calculated from distance and length measurements.

Procedure

Place a long vertical rod into the rod-holder furthest from the aisle. Erase any markings on your screen and use masking tape to mount it onto the vertical rod. With the balancing frame assembly removed from the stationary base, position the base near the other end of the countertop with the frame-bearing posts about 1.5 meters away from the screen. (EXCEPTION: If there is a ceiling vent nearby that creates a draft on your current balance, then relocate it to a draft-free location, as it is very sensitive to air currents.) Adjust the leveling screws to position the base firmly. (See top photo on p.9.)

Carefully replace the frame and position it on the bearing posts using the lifting arms. Make sure the damping vane does not rub against the damping magnets or the edges of the slot in the base when the conducting rod of the frame pivots up and down. If it does, make adjustments and reposition the balance frame using the lifting arms. Make sure it moves freely with no rubbing

NOTE: PROTECT THE BEARING SURFACES AND PIVOT EDGES FROM ACCIDENTAL SCRATCHING OR DENTING. USE ONLY THE LIFTING ARMS TO REPOSITION THE BALANCE ASSEMBLY BEARING EDGES SOFTLY AND REPEATABLY ONTO THE POSTS.

To check the two conducting rods for alignment and distortion, place a coin on the scale pan to bring them into contact. Thumbscrews on the frame, above each pivot edge, permit either end of the upper rod to be moved forward or backward until it is parallel to the bottom rod (when looking down). The two rods should be aligned as accurately as can be determined by the unaided eye when viewed from the above. Raise and lower the balance assembly with the lifting arms after every adjustment to make sure alignment is repeatable. View the rods from the side. If the rods are not touching at both ends, use the thumbscrew on the front post nearest the end of the bottom conducting rod that's not touching to raise it until both ends are touching. **The importance of precise alignment cannot be overemphasized.** When viewed from the front, with a white paper behind the rods, either of the two rods may appear to be slightly bowed. If this is significant, correct by gently bending one rod or the other by hand until both appear to be straight. It is difficult to get them so straight that no light may be seen between them, but this is not essential for good quantitative results. In general, the rods should always be handled gently.

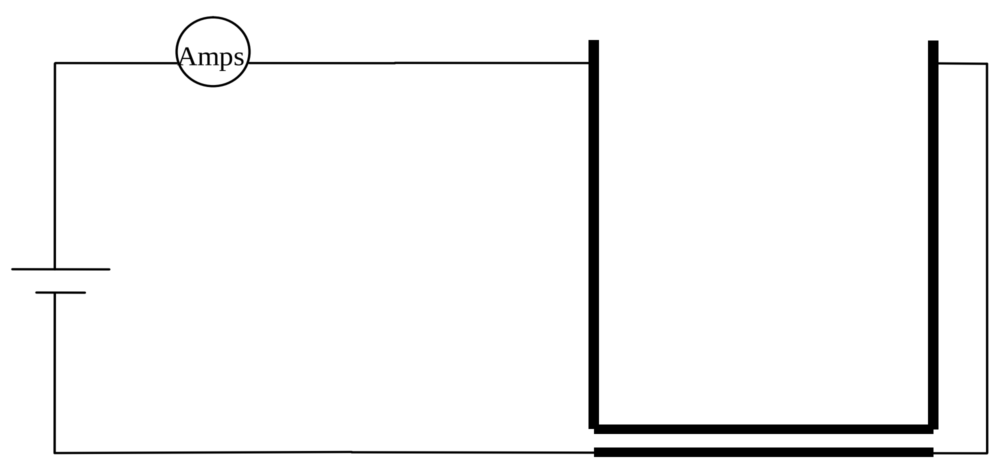


Fig. 4 Schematic of Circuit to be Constructed for Repulsion Between Rods

Setup the simple circuit as shown in Fig. 4 (above). The voltage source is the variable DC power supply provided. (A photograph of the setup described in Fig. 4 is provided on the bottom of page 9.) The circuit depicted in Fig.4 provides the configuration for a repulsive force (currents flowing anti-parallel) between the two rods as required in the experiment. Do <u>not</u> turn the power supply on yet.

With the coin still on the balance weight pan, rotate the lifting arm bar to raise the balance frame off the bearing posts and very gently lower it back down until the lifting arms are free of the balance frame. Setup the laser about midway between the mirror and the screen such that the laser beam reflects from the central part of the mirror onto a suitable place on the screen. You will need to use a roll of masking tape or books to aim the laser to do this (See photo on p. 9.). Use your hand to find and block the reflected beam until it's on the screen.

Adjust the counterpoise below the mirror to the lowest position. Gently reposition the balance frame with the lifting arms and check that the laser dot is still suitably located on the screen.

Remove the coin. Patiently adjust the counterpoise weight behind the mirror until the upper conducting rod comes to rest at an equilibrium position about one millimeter above the stationary bar (Read below first).

<u>TO EXPEDITE GETTING THE UPPER ROD TO SETTLE AT ITS EQUILIBRIUM POSITION THE FOLLOWING TECHNIQUE IS SUGGESTED</u>: CAREFULLY PLACE YOUR THUMB (OR FINGER) ON THE LOWER ROD AND SLOWLY "ROLL" IT UPWARDS UNTIL IT TOUCHES THE OSCILLATING UPPER ROD WHICH WILL PROBABLY BOUNCE OFF IT. <u>VERY</u> SLOWLY ROLL YOUR THUMB JUST HIGH ENOUGH THAT THE UPPER ROD COMES TO REST ON IT, THEN, EVEN <u>MORE SLOWLY</u>, ROLL YOUR THUMB DOWNWARD UNTIL THE UPPER ROD IS NO LONGER RESTING ON IT, BUT IS RESTING (OR WITH VERY LITTLE MOTION) AT ITS EQUILIBRIUM POSITION.

Raise and gently lower the balance frame again and make sure that the equilibrium separation between the conducting rods is still about 1 mm.

Carefully place the coin back on the weight "pan" (without touching the balance), so the current balance rods are aligned and touching again. Mark the position of the center of the laser beam dot on the screen with a line (in pencil), and label it "T", or "touching". This mark will be the zero reference for subsequent measurements on the screen.

Carefully remove the coin from the weight pan using the tweezers from your weight set box. The frame should oscillate freely about its equilibrium position (i.e. about 1 mm space between rods). If necessary, use the technique previously described to hasten settling of the upper rod at its equilibrium position. Mark the equilibrium position of the center of the laser dot on the screen and label it "E" or "equilibrium". If the laser dot doesn't come to nearly complete rest, estimate the midpoint of the continuing oscillations by holding a pencil point on the screen and seeing if the dot goes equally above and below it. If it doesn't, make a better estimate and repeat the process until you have an accurate estimated equilibrium location to mark and label.

IF THE LASER OR THE BALANCE IS ACCIDENTALLY JARRED DURING THE SUBSEQUENT MEASUREMENT YOU WILL HAVE TO GO BACK TO THE PREVIOUS STEPS AND REESTABLISH YOUR TOUCH AND EQUILIBRIUM MARKS AND START OVER, <u>SO BE CAREFUL!</u>

Turn the power supply on and set the current to maximum (clockwise) setting, then adjust the voltage setting to attain the greatest current (amperage) reading possible on the power supply display. (You should be able to get at least 10 amps.) Leave the voltage knob at this setting and turn the current knob counterclockwise to zero. TURN THE POWER SUPPLY OFF.

Caution: At high current settings periodically check the wires for heating. If they are getting too hot, turn the current down and the power supply off and let them cool to a safe temperature.

Begin a trial by adding a 100 mg mass to the pan (use tweezers). Turn the power supply back on and slowly increase the current setting until the center of the laser beam dot returns to its equilibrium position. If this is not possible, start with a 90 mg mass (or lower, if necessary) instead. Record the weight of the mass and the equilibrium current in the Data Table on p. 8. Set the current back to zero, place 90 mg (or 10 mg less than your starting mass) on the weight pan and again adjust the current until the center of the laser dot settles at the equilibrium location. Record the weight and the equilibrium amperage reading in the Data Table. Repeat this process, decreasing the mass by 10 mg increments, down to 10 mg.

Engage the beam lift gently and reposition the frame; then gently release it and again locate the equilibrium point. If it deviates significantly from the first observation, the base may be unsteady, or the balance or laser may have been jarred during measurements and you will have to start over.

Remove the balance assembly from the base to make measurements: (Assign uncertainties to them!)

- Measure the horizontal distance from the mirror to the screen and record this number as b in the top data table on p.7.
- The separation between the centers of the two rods when they are touching is the sum of their radii, r_1+r_2. Measure these radii with a caliper and record their sum in the data table.
- Measure the distance from the pivot edge (on the bearing post) to the axial centerline of the top parallel rod on each end and record the average of these two as "a" in the data table.
- Measure the length of the top parallel rod to be used in the calculation of the force. Record this value as "L" in the data table.
- The screen marking at contact "T" has been chosen as the zero reference. Measure the distance between this mark and the equilibrium mark "E" and enter it as D, the "screen reading at equilibrium", in the data table.

Because the angle of reflection of the laser beam must equal the angle of incidence, when the balance assembly, and hence the mirror, rotates through an angle θ, the reflected laser beam rotates through an angle 2θ. For small mirror angle changes, the distance the laser dot moves on the screen is well approximated as the product of the distance between the mirror and the screen, b, and the angle, 2θ (in radians), so we set $D = 2b\theta$. Similarly, the corresponding separation distance between the surfaces of the two rods is $d_o = a\theta$. Taking the ratio of these two equations and solving for d_o yields: $d_o = (a/2b)D$. Calculate d_o and record it in the data table.

The centerline-to-centerline distance, d, between the current-carrying rods is the sum of d_o and the two radii of the current-carrying rods, i.e. $d = d_o+r_1+r_2$. Calculate d and record it in the data table. (Note: "d" will be used as “*r*” in equation (7) since the locations of the two currents are assumed to be concentrated entirely along the centerlines of the rods in our theoretical model.)

Use equation (7) from the theoretical model described at the beginning of the lab, to calculate the slope predicted by this model. Use r = d, your measured value of L and $\mu_o = 4\pi \times 10^{-7}$ N/A^2. Record these data in data table.

Plot the force (i.e. weight = mg, in Newtons!) versus the current squared. Perform a straight line fit and determine the slope of the line and the coefficient of correlation, r. Record these in the data table.

Data Tables

	Trial 1	**Trial 2 (Optional)**
Mirror to scale, b		
Sum of radii of rods, r_1+r_2		
Lever arm, a		
Length of upper bar, L		
Screen reading at contact	0	0
Screen reading at equilibrium, D		
Separation of between rod surfaces at equilibrium, d_o		
Rod centerline separation at equilibrium, $d= d_o+ r_1+r_2$		
Predicted slope from theory and length measurements		
Slope from least squares fit of F vs. I^2		
Correlation coefficient of linear plot of F vs. I^2		

Trial 1:

Mass (mg)	**Force , mg (N)**	**I(A)**	**$I^2(A^2)$**

Following Up

Assign uncertainties to your measurements and make estimates of the uncertainties in calculated values.

Was the predicted linear dependence of F on I^2 well verified by your data? Explain.

Was the predicted numerical value of slope of F vs. I^2 well verified by your data? Explain.

Was the predicted intercept of F vs. I^2 well verified by your data? Explain.

If the linear dependence, slope or intercept inferred from your experimental data appear to be in significant disagreement with predictions calculated using equation (7), provide a reasonable guess as to why.

Using a value of the slope determined experimentally from your plot of F vs. I^2 (as well as the measured value of "L"), determine what the value of "d" (i.e. "r") would have to be in equation (7) in order to have agreement with theory. Let's call this value the "effective distance" and label it "d_{eff}".

Recall the corresponding value of "d" calculated from distance and length measurements made in the Procedure section (i.e. using $d_o = (a/2b)D$, and $d = d_o + r_1 + r_2$). Let's call this the measured value of d and label it "d_m". Is d_{eff} smaller or larger than d_m?

Calculate the % difference between these two different values of "d" using the latter as reference.

Lab Report

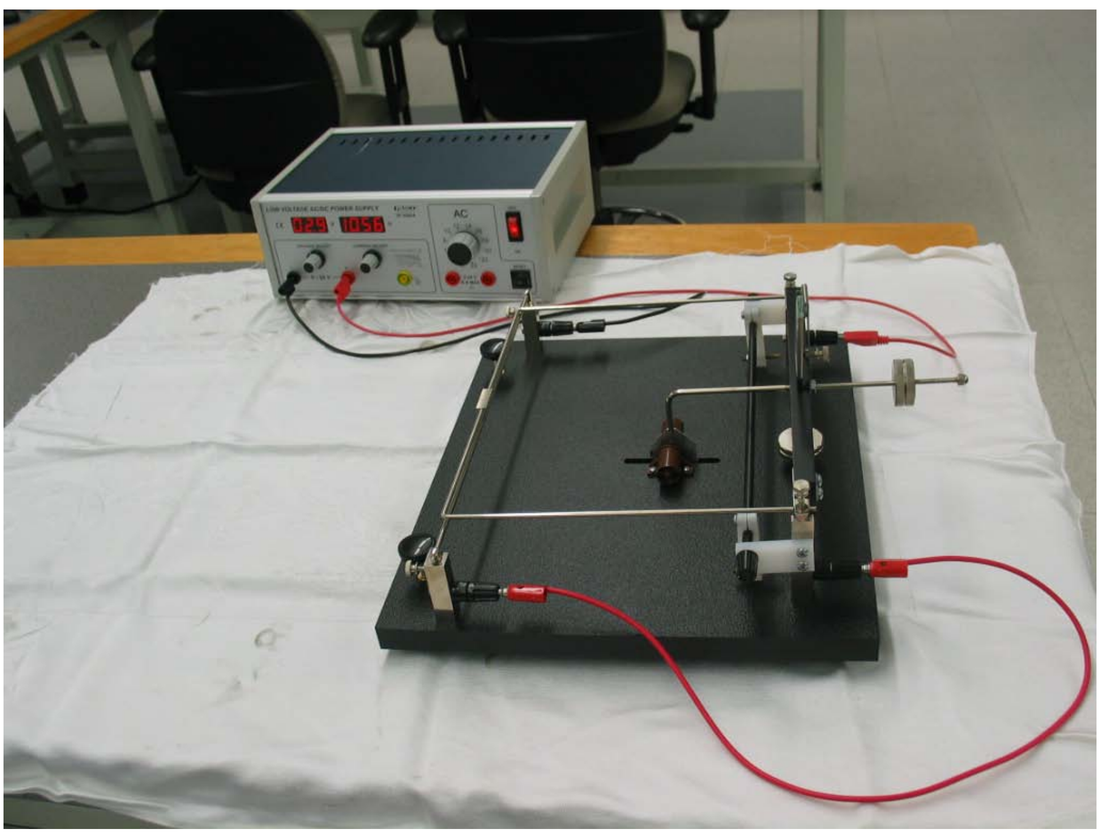
Summarize your results, provide samples of all calculations and state your conclusions. In your summary be sure to discuss the suitability of the theoretical model used (i.e. equation (7)) for comparison with your experimental results. If either the linear dependence or the value of the slope inferred from your experimental data disagrees significantly with predictions calculated using equation (7), provide a well reasoned explanation, supported by data, why.

OPTIONAL (If time permits): For a second trial, change the counterpoise slightly to make the separation of the conducting rods slightly different. Mark the laser dot locations at contact and at equilibrium, and take another series of readings. Record your readings in the Data Tables and the table below.

Trial 2:

Mass (mg)	**Force , mg (N)**	**I(A)**	**$I^2(A^2)$**

By eq. (7), the ratio of d_2 (trial 2) to d_1 (trial 1) should equal the inverse ratio of their slopes (i.e. $d_2/ d_1 = m_1/m_2$). Do your results confirm this relation reasonably well? If not, try to explain why.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