Worksheet: Electromagnetic Induction (revised 1/11/11)

***NAME*____**

Changing magnetic flux through any closed loop creates an induced *emf* in that loop. The magnetic flux can change either because the magnetic field changes, the area of the loop changes, or because the angle between the area and the field changes. The following cases illustrate some examples.

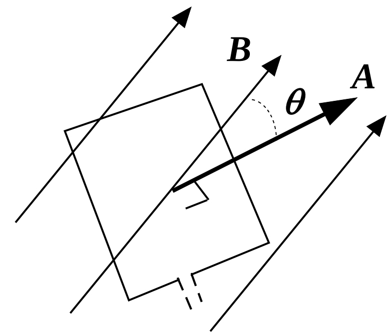


Figure 1: Changing Magnetic Flux creates an induced *EMF*

Induced *EMF*: $\mathcal{E} = -\frac{d\Phi_B}{dt}$ Current $I = \frac{\mathcal{E}}{R}$

Magnetic Flux: $\Phi_B = \int \vec{B} \bullet d\vec{A} \quad = \vec{B} \bullet \vec{A}$ for uniform field

$$\vec{B} \bullet \vec{A} = |B||A| \cos\theta_{BA}$$

$$-\frac{d\Phi_B}{dt} = -\left(|A| \cos\theta_{BA} \frac{dB}{dt} + |B| \cos\theta_{BA} \frac{dA}{dt} - |B||A| \sin\theta_{BA} \frac{d\theta_{BA}}{dt} \right)$$

Problem 1: A magnetic field of 0.30 T goes through a circular loop of radius R=10 cm so that it makes an angle of 30 degrees with the area vector.

What is the magnetic flux through the loop?

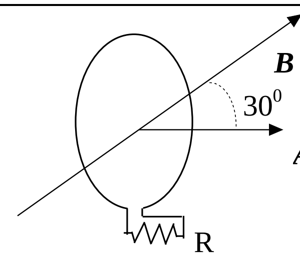


Figure 2: Magnetic Field (B) is at a 30 degree angle to the area vector.

$\Phi_B =$	Weber=T m^2 (units of mag. flux)

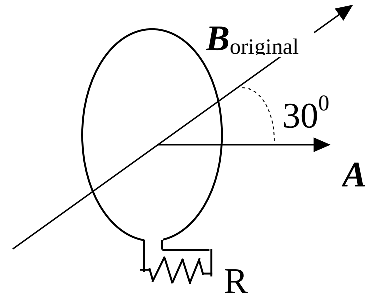
The magnetic field *increases* from 0.30 T to 0.50 T in one tenth of a second. If we assume that it increases at a constant rate, what is that rate? (Rate = change per unit time)

$\frac{dB}{dt} =$

What is the induced *EMF* created in the loop? If the resistance in the loop is 20 Ohms, what is the induced current?

$\mathcal{E} =$
$I =$

Lenz' Law: the direction of the induced *EMF* is such that the induced currents will produce a magnetic field which tends to counter-act the changing magnetic flux.



This means that in the previous example, since the magnetic field and therefore the flux was *increasing*, the induced current will tend to counter-act that change by creating a magnetic field in the *opposite* direction of the original field (thus subtracting from the increasing flux). Use the right hand rule to find the direction of that current, ***I***¸ and the direction of the induced magnetic field, B_{induced}, created by the induced current. Sketch ***I*** and B_{induced} above.

<u>Problem 2:</u> A long solenoid (cylinder wrapped with many turns of wire) creates a magnetic field *inside* of it equal to $B = \mu_0 n I$, where $\mu_0 = 4\pi \times 10^{-7}\, T \cdot m / A$, n is the number of turns of wire per unit length, and *I* is the current in the wire. If the solenoid is very long, then outside (far from the ends) the field is zero.

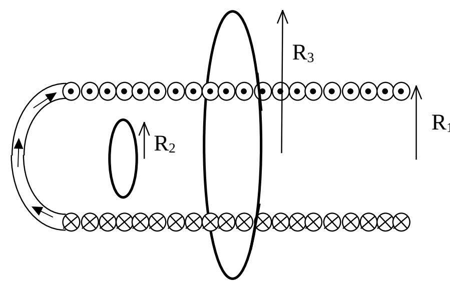


Figure 3: Large loop outside a solenoid

Consider a solenoid with 100 turns of wire per cm (n=100 cm^{-1}). If a current of 2A runs through it, what is the field inside? (Be careful of your units! You must convert turns/cm to turns/m.)

***B*=**

Let the radius of the solenoid be R_1=2 cm, the radius of a small loop inside it be R_2=1 cm and the radius of a big loop outside it be R_3=5cm. Find the flux through the two loops. (Be careful! Remember the field outside the solenoid is zero, so there is only flux through part of the area of the outer loop.)

Small Loop: $\Phi_B =$
Large Loop: $\Phi_B =$

Now suppose that the current is decreased at a constant rate from 2A to zero over a time of 0.2 seconds. What is the *EMF* induced in (around) each of the two loops (if there were breaks in them)?

Small Loop: $\mathcal{E}_2 =$
Large Loop: $\mathcal{E}_3 =$

Problem 3: A circuit consists of three fixed metal rails (shown in black) on which a fourth rail (shown in white) is moving at constant speed v_0.

A constant magnetic field of magnitude *B* comes out of the paper.

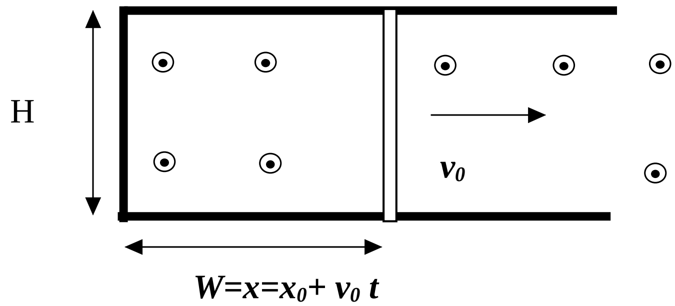


Figure 4: The area increases as the bar travels along the rails.

The area inside the closed circuit is A=W x H. Show that $\frac{dA}{dt} = H \cdot v_0$

(*What is constant? What is changing with time?)*

If the black rails have zero resistance and the white rail has resistance *R*, show that the magnitude of the current in the rail is: $I = BHv_0 / R$.

The direction of the current should be such as to counter-act the changing flux. Is the flux increasing or decreasing? Should the induced current add to the original *B* field or subtract from it? (Hint: another way to tell the direction of the induced current is that the force on the white rail, $\vec{F} = I\vec{L} \times \vec{B}$, should also be in a direction which opposes the changing flux by trying to stop the change.) Draw the direction of the current on the figure.

Problem 4: An AC (Alternating Current) generator can be made by rotating a current loop in a magnetic field. If the loop of area A rotates at angular frequency ω then we can show that the flux changes (because the angle between ***A*** and ***B*** changes) so that:

$$-\frac{d\Phi_B}{dt} = -|A||B|\frac{d}{dt}\cos\theta = |A||B|\sin\theta\frac{d\theta}{dt}$$

The induced *EMF* thus varies sinusoidally with time: $\mathcal{E}(t) = V_{\max}\sin\theta = V_{\max}\sin(\omega t)$

We wish to generate AC voltage with amplitude $V_{\max}$=170 Volts at a frequency of $f = \omega / 2\pi$ =60Hz. If we have a magnet which can create a field of *B*=0.25 T, what area of loop will we need? What area would we need if there were 100 turns of wire in the loop?

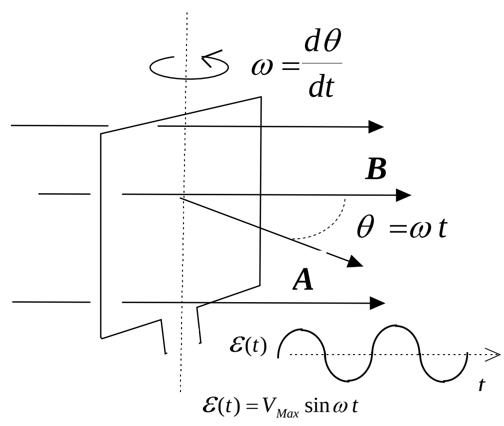


Figure 5: A loop of wire rotates in a magnetic field.

A=	A(100 turns)=