

## Millikan Oil-Drop Experiment

Electrons were observed around the turn of the century by J.J. Thomson when he measured the ratio of the electron's charge to its mass ( $e/m$ ). However, Thomson's experiment only gave a value for the ratio  $e/m$ . The charge of the electron was measured by Millikan in a now classic experiment performed by students in Modern Physics classes around the world.

In the Millikan experiment, a small charged sphere made of latex moves vertically between two metal plates. This sphere is too small to be seen with the naked eye, and so a projector and microscope system are used to enable the user to see the sphere as a small dot of light.

When there is no voltage applied, the sphere is accelerated downward by the force of gravity. Yet, when moving, the air resistance leads to an opposite force,  $F_{\text{stokes}}$ , which is proportional to the speed. The value of the air resistance force on a sphere was first derived by Sir George Stokes and is given as

$$F_{\text{Stokes}} = 6\pi \eta r v$$

where  $\eta$  is the **viscosity** of the medium,  $r$  is the **radius** of the sphere, and  $v$  is its **speed**. This equation is known as *Stokes' law*. After a short time, the force of gravity and the force of air resistance will become equal and the sphere reaches its **terminal speed**.

Consider a latex sphere of mass  $m$  and charge  $q$  between two horizontal plates. When there is no voltage applied to the plates, the sphere falls slowly and steadily under the influence of gravity, quickly reaching its terminal speed,  $v_0$ , at which time the weight of the sphere,  $mg$ , is exactly equal to the air resistance force. This situation is similar to that of a parachutist falling through air. When a voltage is applied to the plates, the terminal speed of the sphere is affected not only by the force of gravity but also by the electric force acting on the sphere. The ball will reach a higher terminal speed,  $v_d$ , when the field is oriented down. When the field is oriented up, the ball will move upward and reach a terminal speed,  $v_u$ .

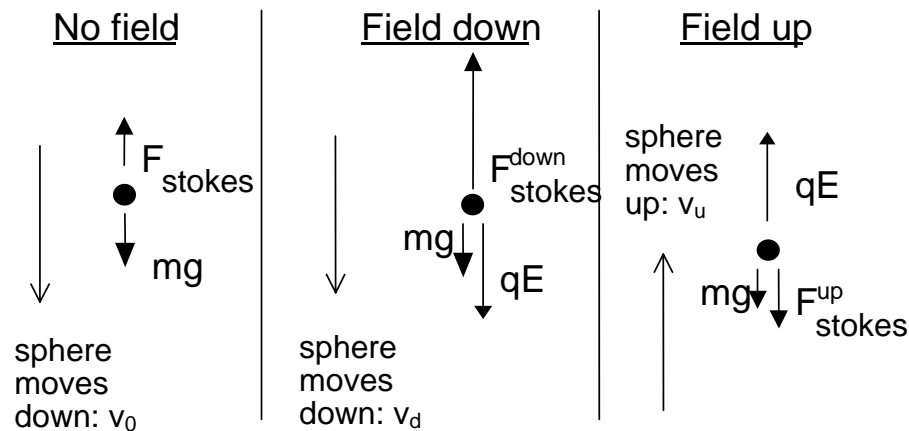


Figure: Force diagram for the sphere for the case of no field, field downward and field upward

When the experimenter knows the density of the latex ball, the terminal velocity of a ball falling under the influence of gravity alone, the terminal velocities of the sphere when the fields are applied, and the charge on the plates of the Millikan apparatus, then it is possible to determine the elementary unit of charge. (This unit of charge is known as the electron charge ( $e$ ) and has the value of  $1.602 \times 10^{-19}$  coulomb).

### No Field

The force diagram shows that the weight of the sphere,  $mg$ , is exactly equal to the air resistance force.

The equation of motion of the sphere (Newton's 2<sup>nd</sup> law) without any field applied is:

$$mg = F_{\text{Stokes}} = 6\pi\eta r v_0$$

The mass of the sphere is given by

$$m = \rho V = \frac{4}{3}\pi r^3 \rho$$

**The radius of the sphere  $r$**  can be calculated from the measured terminal speed  $v_0$ , when the two formulas above are combined and the following values are taken into consideration:

For the latex spheres in air:  $\eta = 1.816 \times 10^{-5}$  kg/(ms);

Gravitational constant:  $g = 9.81$  m/s<sup>2</sup>

Density of latex:  $\rho = 1.05$  g/ml

$$r = \sqrt{\frac{9v_0\eta}{2g\rho}} = 8.91 \times 10^{-5} \sqrt{v_0}$$

Hereby  $v_0$  is in m/s. A typical value of  $r$  is  $6 \times 10^{-7}$  m.

### Field Up / Field Down

Now suppose the metal plates are connected to a source of constant potential difference such that an electric field of intensity  $E$  is established between the plates and that a latex sphere of charge  $q$  is made to move upwards. The direction of the electric force must depend on the sign of the charge  $q$ , which may be either positive or negative.

The resulting upward force on the charge is  $qE - mg$  and this force causes the sphere to move upwards with a constant terminal speed,  $v_u$ . The equation of motion is

$$qE - mg + 6\pi\eta r v_u = 0$$

If the polarity of the electric field is reversed, the sphere will move downwards under the combined force of gravity and the electrostatic force. This equation of motion is

$$qE + mg - 6\pi\eta r v_d = 0$$

Notice that the forces are now additive and that the terminal speed is achieved in the opposite direction than in the previous case.

The effects of gravity can now be eliminated by subtracting the equations of motion:

$$2qE - 6\pi\eta r (v_u + v_d) = 0$$

$$qE = 3\pi\eta r (v_u + v_d)$$

Both  $\eta$  and  $r$  are known quantities, which will allow the calculation of the charge  $q$ , when the electrostatic field  $E = V/D$  ( $D$  distance between plates;  $D = 5 \times 10^{-3}$  m;  $V$  applied voltage) and the terminal speeds  $v_u$  and  $v_d$  are

measured.

## Procedure

Begin by setting the apparatus on a level surface. Make sure all power to the unit is turned off whenever you are making adjustments to it. [If necessary, unscrew the electrode housing set screws, remove the upper electrode housing plate, and then insert atomizer ring between the upper and lower electrode housing; retighten set screws].

1. Ensure that the D.C. volts leads are plugged into their respective color terminal plugs.
2. Position the 3-way polarity switch in its mid position, establishing a “no charge” condition. Turn the “on/off” switch to “on.” The illuminating lamp should light.
3. Adjust the microscope by rotating the focus adjusting knob until an approximate mid position is established. The eyepiece divisions should be distinguishable from the background.
4. Adjust the electrode voltage to ~100 volts using the voltage adjusting knob, but keep the polarity switch in its mid position.
5. Spray latex into the apparatus and carefully look for the “dots of light” in the microscope. If after several pumps on the atomizer bulb, the latex spheres are still not visible, adjust the microscope carefully to attempt to focus in on the spheres.
6. Get accustomed to the movement of the dots, by switching on the plate voltage for a short period of time (both directions). Obtaining a suitable drop may require patience, for drops continue to enter the region between the plates for several seconds after spraying has stopped.
7. Choose one particle, which drifts slowly down with no voltage and slowly up with the voltage in supporting the up-direction (about 30 s for 1-2 large divisions of the eyepiece). It is desirable for one person to observe the droplet, handle the stopwatch and tell the moved distance and a second person to read the stopwatch and note the distances and times.
8. Record the fall time with no field  $t_0$ , the rise time  $t_+$  and the fall time  $t_-$  with field **of the same latex sphere** and note as well the respective distances in small lines for each measurement (4 small lines are 1mm). Make at least two measurements for each field setting (field off, up, down) – always with the same latex sphere.
9. Calculate the corresponding speeds  $v_0$ ,  $v_u$  and  $v_d$ , which are merely the distance (in mm) divided by the time the sphere took to travel that distance.

Each person of the group has to take (at least) one set of data for his/her latex sphere.

In the write-up, note your plate Voltage  $V$ , the speed  $v_0$ ,  $v_u$ , and  $v_d$ . Additionally, add the calculated values of the sphere radius  $r$ , and the charge  $q$ . The radius of the sphere can be calculated from the terminal speed  $v_0$  (in m/s) as described in the theory part:

**The charge  $q$**  is derived from the average terminal speed  $\{(v_u + v_d)/2\}$ . The actual value of the charge can be calculated by:

$$q = 6\pi \eta r \{(v_u + v_d)/2\} D / V$$

with  $D$  as the distance between the plates,  $V$  as the applied voltage, and  $\eta$  as the air viscosity.

For the latex spheres in air:  $\eta = 1.816 \times 10^{-5} \text{ kg/(ms)}$ ;

Distance between the plates:  $D = 5 \times 10^{-3} \text{ m}$

It is important to realize that the sphere can have only integer multiples of the elementary charge:  $q = ne$ . With  $n=1,2,3,4\dots$  Many measurements have to be conducted and the results compared. A final interpretation can only be made after many data points have been collected. It is expected (results of Millikan) that the data points fall into groups representing spheres with the same charge. These groups are each separated in equal integer distances of  $e$  ( $1.6 \times 10^{-19} \text{ C}$ ). [first group: values around  $e$ , 2<sup>nd</sup>: values around  $2e$ ; 3<sup>rd</sup>: values around  $3e$ ]. The experiment leads to two findings: The quantization of charge (discrete steps) and the actual value of  $e$ .

### Data for one and the same sphere:

Voltage V: \_\_\_\_\_

**Table 1:** No Field

#	Time (sec)	# Small Lines	$V_0$
1			
2			
3			

**Table 2:** Field Up

#	Time (sec)	# Small Lines	$V_u$
1			
2			
3			

**Table 3:** Field Down

#	Time (sec)	# Small Lines	$V_d$
1			
2			
3			

NOTE: 4 small lines correspond to 1mm.

Radius r: \_\_\_\_\_

Charge q: \_\_\_\_\_

### Questions:

- Without any field applied, how does the terminal speed depend on the size of the particle?
- Are the spheres positively or negatively charged? Explain.
- What is the percent difference between your value and the known value of  $e = 1.6 \times 10^{-19} \text{ C}$ ?
- What are the uncertainties in your measurement. Which one is the most significant? What could be done to improve the results of the experiment? (To what percent can you expect your value for  $e$  to be good)?

IMPORTANT: For the write-up include, title of experiment, purpose and small description of the procedure. Summarize the results in a table (time, distance, calculated speeds, radius) and address the questions mentioned

above. Insert your points in the Millikan Oil Drop Result Sheet.