Hooke's Law and Simple Harmonic Motion (approx. 2 hr) (7/20/11)

Introduction

The force applied by an ideal spring is governed by *Hooke's Law*: F = -kx. Because the force is proportional to displacement of the spring from its equilibrium position, a mass attached to the spring will undergo *simple harmonic motion.* In this lab we will verify Hooke's Law and learn about simple harmonic motion. The motion of the spring will be compared to motion of a pendulum. Damped harmonic motion may also be investigated.

Equipment

- spring
- motion sensor
- meter stick
- vertical rod (51")
- masking tape
- pendulum clamp
- cardboard dampener
- pendulum bob
- bench-edge clamp
- keyhole mass set
- keyhole mass hanger

For class as a whole: beam balance

Theory and Overview

Hooke's Law

The first part of this experiment is designed to verify Hooke's law. For a mass hanging from a spring, Hooke's law states that the force, F, applied to the hanging mass is proportional to the displacement, $(x-x_0)$, of the mass from its equilibrium position without the mass, that is,

$$F = -k \cdot \left(x - x_0\right),$$

where k is the spring constant. The minus sign occurs because the force is opposite to the direction of displacement. Hooke's law can be used to calculate the spring constant, k, if the force and displacement from equilibrium resulting from it have are known.

Simple Harmonic Motion

In the second part of the lab we will verify that motion under this force is "simple harmonic motion" with a period, *T*, of

$$T = 2\pi \cdot \sqrt{\frac{m}{k}}$$

Thus, *k* can also be determined by measuring the mass and its period of oscillation.

Small Oscillations of a Simple Pendulum

In the third part of the lab we will verify the theoretical prediction that a simple pendulum (i.e. mass hanging from a string) should exhibit simple harmonic motion for small oscillations and we investigate the dependence of the period of a simple pendulum on its length.

Damped Oscillations

The forth part of the lab provides an opportunity to investigate the effect of a damping force (in this case, air drag) on simple harmonic motion.

Procedure

Part 1: Verifying Hooke's Law and Measuring the Spring Constant

In this part, the DataStudio program is used with the motion sensor to measure the displacement of different masses. If x_0 is the equilibrium position of the spring (no masses added), then the displacement is $(x\text{-}x_0)$, where "x" is the position measured by the motion sensor (see Figure 1). According to Hooke's Law, the force applied by the spring is:

$$F(x) = -k \cdot \left(x - x_0\right) = -k \cdot x + k \cdot x_0 .$$

Graphing software will be used to make a plot of F versus x. Hooke's law is verified if F vs. x is a straight line. The slope of this line represents the spring constant k (the intercept will be kx_0).

Data collection

DO NOT LET MASSES DROP ONTO THE MOTION DETECTOR (secure in place)

In this part you will use the motion sensor to measure distances for stationary masses.

Set up a spring-mass system as shown in Figure 1 with the spring hanging from the end of the pendulum clamp beyond the edge of the table and the motion sensor on the floor. First, use your pendulum bob as a plumb bob to position the motion sensor directly beneath the mass hanger.

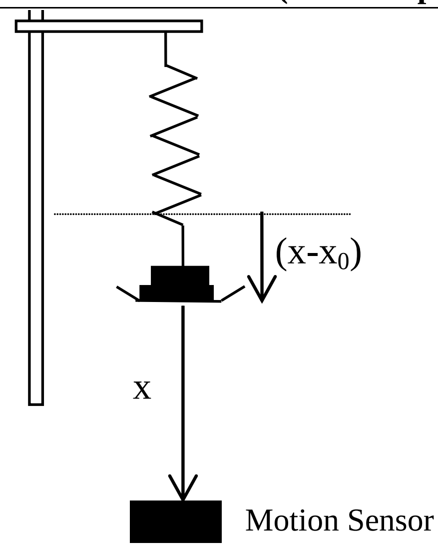

Figure 1: Mass on Spring with Motion Sensor.

On your computer desktop, click on <u>DataStudio</u> and <u>Create Experiment.</u>

In the Experimental Set-up window choose <u>Motion Sensor.</u>

Connect the motion sensor to the interface as shown, set it on the narrow beam setting, and set the sensor trigger rate to 50. (Double click on the displayed sensor icon attached to the interface.)

To create a graph, double click on the "Graph" icon in the "Displays" menu and choose "Position" as the Data Source for your graph.

Clicking **Start** and **Stop** will start and stop the motion sensor. Make sure the position you are measuring is the distance to the mass hanger on the spring by giving it a slight vertical motion. (You can also use your meter stick.) By letting the software run while the mass is stationary you will be able to get average position readings for different suspended masses.

First, measure the position of the mass hanger with the maximum (500g) mass on it. Click **Start** to collect data. The position as a function of time should be a horizontal line. Click **Stop** to stop the motion sensor. Estimate position by reading the graph's axis. (Note: You can change the scale of the axis by clicking and dragging on the axis.) Record the position in Table 1.

Repeat the measurement of position for different masses by placing the listed masses (Table 1) on the hanger, and recording the mean position values.

Express F in terms of Newtons and fill out the last column in Table 1.

Input F and x from Table 1 into a graphing program. Make a plot of F vs x. F vs. x is expected to be linear with slope corresponding to the spring constant k. Determine the slope by using a linear fit (trend line). If your plot does not seem to fit a straight line it may be because your masses are too close or too far from the sensor at one end or the other.

Part 2: An Oscillating Mass Undergoing Simple Harmonic Motion

This part is designed to verify that the motion of the oscillating mass on the spring is simple harmonic motion with a period of

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(Equation 1)}$$

Because of the mass contribution from the spring, m in this equation consists of the mass of the suspended weight m_W and the effective mass of the spring M_e. The above equation can be written as

$$T = 2\pi \sqrt{\frac{m_W + M_e}{k}}\ , \qquad \text{(Equation 2)}$$

or, by squaring both sides:

$$T^2 = \frac{4\pi^2}{k} m_W + \frac{4\pi^2}{k} M_e \qquad \text{(Equation 3)}$$

Equation (3) suggests that T^2 vs. m_W is linear with slope of $\frac{4\pi^2}{k}$ and intercept equal to $\frac{4\pi^2}{k} M_e$. Thus, both the spring constant, k, and the effective mass of the spring, M_e , can be determined from the T^2 vs. m_W graph. The k derived in this method should be very close to the one obtained by the Hooke's law method. The ratio of M_e to the actual mass of the spring, M_s , is always less than one.

Data collection

The same set-up is used in this part.
Add 100 grams to the hanger.

1. Give the mass a small initial displacement from equilibrium and release it.
2. Click **Start** to collect data. The position as a function of time should look like a sine function. (That is precisely what is meant by simple harmonic motion!)
3. Click **Stop** to stop the motion sensor.
4. Change the scale of your vertical axis (click and drag on the numbers) until you see your graph well. If your data does not look like a nice sine wave you may need to adjust the alignment of your apparatus or the data collection (i.e. trigger) rate.

Repeat steps 1-4 with different weights as listed in Table 2, and record times to Table 2.
Calculate T and T^2 (See Fig. 2 below.) and enter them in Table 2.
Input T^2 and m_W to your graphing program. (Don't forget to convert grams to kilograms!)
Make a plot of T^2 vs. m_W . Find the slope and intercept from your graph.
Calculate the spring constant k and compare it with the value obtained in Part 1. Calculate M_e.
Measure the mass, M, of your spring. Calculate the ratio M_e/M.

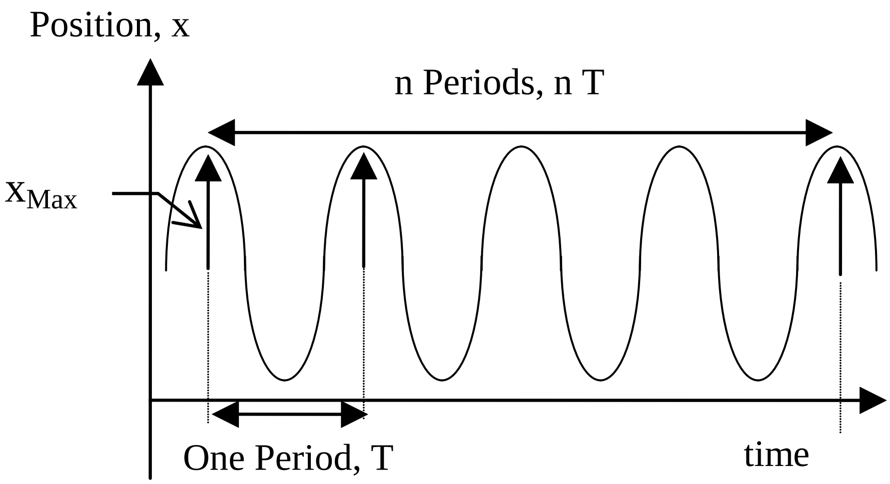

Figure 2: Simple harmonic motion: the mass oscillates like a sin function: $x(t) = x_{Max} \sin\left(\frac{2\pi}{T} t\right)$

Note: For the above example the number of oscillations (full cycles) is n=4.

Part 3: Small Oscillations of a Simple Pendulum

In this part, we verify an expression for the period of a pendulum. For *small angles* of oscillation, the torque on the pendulum bob is proportional to the angle of displacement. This means that the pendulum should also undergo simple harmonic motion. We shall measure the period of oscillation and find out how it is related to the length of the pendulum (i.e. the distance from the attachment point of the string to the center of mass of the pendulum bob).

Data Collection:

Set up a simple pendulum and position the ultrasonic motion sensor to monitor its horizontal motion. Position the sensor close enough to the pendulum to get good, smooth graphs.

Displace the bob a small distance from equilibrium in the line of the sensor beam and release it.

Measure the period of the pendulum for several different string lengths (Table 3)

Determine the relationship between the period of the pendulum and its length by plotting both period, T, versus length, and T^2 versus length and seeing which plot best fits a straight line.

Your instructor (or a textbook) can provide the complete theoretical relationship between period and length for small angles of oscillation.

Part 4: Damped Oscillations (optional? – Ask your instructor.)

If a large piece of cardboard with a slot in it is placed on the hanger under the mass in the mass/spring setup, air resistance to the motion of the mass as it oscillates will be greatly increased. Experiment with different masses to get a good position versus time graph that has obviously decreasing amplitude in a reasonably short time. Investigate how this damping force acts to decrease the amplitude of the oscillations over time. Specifically, try to determine if the decrease in amplitude with time can be described reasonably well by a simple function. Support your conclusions with data and graphs.

Table 1: Verifying Hooke's Law

M (mass on the hanger)	Position, x	For a stationary mass, F (= Mg) g=9.8m/s^2
0		
100 g=0.100 kg		
200 g=0.200 kg		
300 g=0.300 kg		
400 g=0.400 kg		
500 g=0.500 kg		

Spring constant from method 1 = ________________________________

Table 2: Oscillation of a Mass on a Spring

m_w (grams)*	t_1	t_2	n	$T = (t_2 - t_1)/n$	T^2
50 + 100					
50 + 200					
50 + 300					
50 + 400					
50 + 500					

*where 50 grams is the mass of the mass hanger.

Spring constant from method 2 = ________________________________

Discussion: (continue on separate pages as needed)

Table 3: Small Oscillations of a Pendulum

Length (meters)	t_1	t_2	n	T $(t_2 - t_1)/n$	T^2
0.2					
0.4					
0.6					
0.8					
1.0					

Discussion (continue on separate pages as needed):